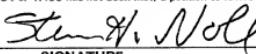


FORM PTO-1390 US DEPARTMENT OF COMMERCE REV. 5-93 PATENT AND TRADEMARK OFFICE		ATTORNEYS DOCKET NUMBER <b>P01.0020</b>
<b>TRANSMITTAL LETTER TO THE UNITED STATES DESIGNATED/ELECTED OFFICE (DO/EO/US) CONCERNING A FILING UNDER 35 U.S.C. 371</b>		U.S. APPLICATION NO. (if known, see 37 CFR 1.5) <b>09/787698</b>
INTERNATIONAL APPLICATION NO. <b>PCT/DE99/02846</b>	INTERNATIONAL FILING DATE <b>08 SEPTEMBER 1999</b>	PRIORITY DATE CLAIMED <b>23 SEPTEMBER 1998</b>
TITLE OF INVENTION <b>METHOD AND APPARATUS FOR DETERMINING A SEQUENCE OF ACTIONS FOR A SYSTEM</b>		
APPLICANT(S) FOR DO/EO/US <b>RALF NEUNEIER ET AL.</b>		
Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:		
<p>1. <input checked="" type="checkbox"/> This is a <b>FIRST</b> submission of items concerning a filing under 35 U.S.C. 371.</p> <p>2. <input type="checkbox"/> This is a <b>SECOND</b> or <b>SUBSEQUENT</b> submission of items concerning a filing under 35 U.S.C. 371.</p> <p>3. <input checked="" type="checkbox"/> This express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay.</p> <p>4. <input checked="" type="checkbox"/> A proper Demand for International Preliminary Examination was made by the 19th month from the earliest claimed priority date.</p> <p>5. <input checked="" type="checkbox"/> A copy of International Application as filed (35 U.S.C. 371(c)(2)).</p> <p>a. <input checked="" type="checkbox"/> is transmitted herewith (required only if not transmitted by the International Bureau).</p> <p>b. <input type="checkbox"/> has been transmitted by the International Bureau.</p> <p>c. <input type="checkbox"/> is not required, as the application was filed in the United States Receiving Office (RO/US).</p> <p>6. <input checked="" type="checkbox"/> A translation of the International Application into English (35 U.S.C. 371(c)(2)).</p> <p>7. <input checked="" type="checkbox"/> Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. §371(c)(3))</p> <p>a. <input type="checkbox"/> are transmitted herewith (required only if not transmitted by the International Bureau).</p> <p>b. <input type="checkbox"/> have been transmitted by the International Bureau.</p> <p>c. <input type="checkbox"/> have not been made; however, the time limit for making such amendments has NOT expired.</p> <p>d. <input checked="" type="checkbox"/> have not been made and will not be made.</p> <p>8. <input type="checkbox"/> A translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).</p> <p>9. <input checked="" type="checkbox"/> An oath or declaration of the inventor(s) (35 U.S.C. 371(c)(4)).</p> <p>10. <input type="checkbox"/> A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371(c)(5)).</p> <p><b>Items 11. to 16. below concern other document(s) or information included:</b></p> <p>11. <input checked="" type="checkbox"/> An Information Disclosure Statement under 37 C.F.R. 1.97 and 1.98; (PTO 1449, Prior Art, Search Report, References).</p> <p>12. <input checked="" type="checkbox"/> An assignment document for recording. A separate cover sheet in compliance with 37 C.F.R. 3.28 and 3.31 is included. <b>(SEE ATTACHED ENVELOPE)</b></p> <p>13. <input checked="" type="checkbox"/> Amendment "A" Prior to Action.</p> <p><input type="checkbox"/> A SECOND or SUBSEQUENT preliminary amendment.</p> <p>14. <input checked="" type="checkbox"/> A substitute specification and substitute specification mark-up.</p> <p>15. <input checked="" type="checkbox"/> A change of address letter attached to the Declaration.</p> <p>16. <input checked="" type="checkbox"/> Other items or information:</p> <p>a. <input checked="" type="checkbox"/> Submission of Drawings</p> <p>b. <input checked="" type="checkbox"/> EXPRESS MAIL #EL 843728583 US dated March 21, 2001.</p>		

U.S. APPLICATION NO. 09/177,876 (10 P-15)		INTERNATIONAL APPLICATION NO PCT/DE99/02846	ATTORNEY'S DOCKET NUMBER P01.00020
17. <input checked="" type="checkbox"/> The following fees are submitted:		CALCULATIONS	PTO USE ONLY
<b>BASIC NATIONAL FEE (37 C.F.R. 1.492(a)(1)(5):</b> Search Report has been prepared by the EPO or JPO \$860.00  International preliminary examination fee paid to USPTO (37 C.F.R. 1.482) \$690.00  No international preliminary examination fee paid to USPTO (37 C.F.R. 1.482) but international search fee paid to USPTO (37 C.F.R. 1.445(b)(2)) \$710.00  Neither international preliminary examination fee (37 C.F.R. 1.482) nor international search fee (37 C.F.R. 1.445(b)(2)) paid to USPTO \$1000.00  International preliminary examination fee paid to USPTO (37 C.F.R. 1.482) and all claims satisfied provisions of PCT Article 33(2)(4) \$100.00			
<b>ENTER APPROPRIATE BASIC FEE AMOUNT =</b>			
\$ 860.00			
Surcharge of \$130.00 for furnishing the oath or declaration later than <input type="checkbox"/> 20 <input type="checkbox"/> 30 months from the earliest claimed priority date (37 C.F.R. 1.492(e)).			
Claims	Number Filed	Number Extra	Rate
Total Claims	21	-20 =	1 X \$ 18.00 \$ 18.00
Independent Claims	02	- 3 =	0 X \$ 80.00 \$
<b>Multiple Dependent Claims</b>			\$270.00 + \$
<b>TOTAL OF ABOVE CALCULATIONS =</b>			
\$ 878.00			
Reduction by 1/2 for filing by small entity, if applicable. Verified Small Entity statement must also be filed. (Note 37 C.F.R. 1.9, 1.27, 1.28)			
<b>SUBTOTAL =</b>			
\$ 878.00			
Processing fee of \$130.00 for furnishing the English translation later than <input type="checkbox"/> 20 <input type="checkbox"/> 30 months from the earliest claimed priority date (37 CFR 1.492(f)). + \$			
<b>TOTAL NATIONAL FEE =</b>			
\$ 878.00			
Fee for recording the enclosed assignment (37 C.F.R. 1.21(h). The assignment must be accompanied by an appropriate cover sheet (37 C.F.R. 3.28, 3.31). \$40.00 per property + \$			
<b>TOTAL FEES ENCLOSED =</b>			
\$ 878.00			
Amount to be refunded \$			
charged \$			
<p>a. <input checked="" type="checkbox"/> A check in the amount of <u>\$878.00</u> to cover the above fees is enclosed.</p> <p>b. <input type="checkbox"/> Please charge my Deposit Account No. _____ in the amount of \$ _____ to cover the above fees. A duplicate copy of this sheet is enclosed.</p> <p>c. <input checked="" type="checkbox"/> The Commissioner is hereby authorized to charge any additional fees which may be required, or credit any overpayment to Deposit Account No. <u>50-1519</u>. A duplicate copy of this sheet is enclosed.</p> <p>NOTE: Where an appropriate time limit under 37 C.F.R. 1.494 or 1.495 has not been met, a petition to revive (37 C.F.R. 1.137(a) or (b)) must be filed and granted to restore the application to pending status.</p>			
<u>SEND ALL CORRESPONDENCE TO:</u> <b>SCHIFF HARDIN &amp; WAITE</b> <b>PATENT DEPARTMENT</b> <b>6600 Sears Tower</b> <b>233 South Wacker Drive</b> <b>Chicago, Illinois 60606-6473</b> <b>CUSTOMER NUMBER 26574</b>			
 <b>SIGNATURE</b> <u>Steven H. Noll</u> <b>NAME</b> <u>28,982</u> <b>Registration Number</b>			

## BOX PCT

IN THE UNITED STATES DESIGNATED/ELECTED OFFICE  
OF THE UNITED STATES PATENT AND TRADEMARK OFFICE  
UNDER THE PATENT COOPERATION TREATY – CHAPTER II

**AMENDMENT "A" PRIOR TO ACTION AND**  
**SUBMISSION OF SUBSTITUTE SPECIFICATION**

APPLICANT(S): NEUNEIER, R., et al.

ATTORNEY DOCKET NO: P01,0020

INTERNATIONAL APPLICATION NO: PCT/DE99/02846

INTERNATIONAL FILING DATE: 8 SEP 1999

INVENTION: METHOD AND ARRANGEMENT FOR  
DETERMINING A SEQUENCE OF  
ACTIONS FOR A SYSTEM

Assistant Commissioner for Patents  
Washington, DC 20231

Sir:

Applicants herewith submit an amendment and substitute specification in  
the captioned PCT application, and respectfully request entry of same prior to  
examination in the United States National Stage.

**IN THE SPECIFICATION**

Cancel the specification as filed and insert therefore the substitute  
specification provided herewith.

**IN THE CLAIMS**

Cancel claims 1 – 21 as filed and insert therefore new claims 22 - 42 as  
follows:

-- What is claimed is:

22. (New) A method for computer-aided determination of a sequence of actions for a system having states, the method comprising the steps of:  
performing a transition in state between two states on the basis of an action;  
determining the sequence of actions to be performed such that a sequence of states results from the sequence of actions;  
optimizing the sequence of steps with regard to a prescribed optimization function, including a variable parameter; and  
using the variable parameter to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

23. (New) The method as claimed in claim 22, further comprising the step of:

using approximative dynamic programming for the purpose of determination.

24. (New) The method as claimed in claim 23, further comprising the step of:

basing the approximative dynamic programming Q-learning.

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25. (New) The method as claimed in claim 24, further comprising  
the steps of:

forming an optimization function within Q-learning in accordance with the  
following rule:

$$\text{OFOQ} = Q(x; w^a),$$

and

adapting weights of the function approximator in accordance with the  
following rule:

$$w_{t+1}^{at} = w_t^{at} + \eta_t \cdot N^k(d_t) \cdot \nabla Q(x_t; w_t^{at})$$

wherein

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{at})$$

26. (New) The method as claimed in claim 23, further comprising the  
step of:

basing the approximative dynamic programming on TD( $\lambda$ )-learning.

27. (New) The method as claimed in claim 26, further comprising the steps of:

forming the optimization function within TD( $\lambda$ )-learning in accordance with the following rule:

OFTD =  $J(x; w)$ ; and

adapting weights of the function approximator are adapted in accordance with the following rule:

$w_{t+1} = w_t + \eta_t \cdot N^k(d_t) \cdot z_t$ , wherein  $d_t = r(w_t, a_t, x_{t+1}) + \gamma J(x_{t+1}; w_t) - J(x_t; w_t)$ ,  $z_t = \lambda \cdot \gamma \cdot z_{t-1} + \nabla J(x_t; w_t)$ , and  $z_{-1} = 0$ .

28. (New) The method as claimed in claim 27, further comprising the step of:

using a technical system to determine the sequence of actions before the determination measured values are measured.

29. (New) The method as claimed in claim 28, further comprising the step of:

subjecting the technical system to open-loop control in accordance with the sequence of actions.

30. (New) The method as claimed in claim 28, further comprising the step of:

subjecting the technical system to closed-loop control in accordance with the sequence of actions.

31. (New) The method as claimed in claim 30, further comprising the step of:

modeling the system as a Markov Decision Problem.

32. (New) The method as claimed in claim 31, further comprising the step of:

using the system in a traffic management system.

33. (New) The method as claimed in claim 31, further comprising the step of:

using the system in a communications system.

34. (New) The method as claimed in claim 31, further comprising the step of:

using the system to carry out access control in a communications network.

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35. (New) The method as claimed in claim 31, further comprising the step of:

using the system to carry out routing in a communications network.

36. (New) A system for determining a sequence of actions for a system having states, wherein a transition in state between two states is performed on the basis of an action, the system comprising:

a processor for determining a sequence of actions, whereby a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, and the optimization function includes a variable parameter for setting a risk which the resulting sequence of states has with respect to a prescribed state of the system.

37. (New) The system as claimed in claim 36, wherein the processor is used to subject a technical system to open-loop control.

38. (New) The system as claimed in claim 36, wherein the processor is used to subject a technical system to closed-loop control. - -

39. The system as claimed in claim 36, wherein the processor is used in a traffic management system.

40. The system as claimed in claim 36, wherein the processor is used in a communications system.

41. The system as claimed in claim 36, wherein the processor is used to carry out access control in a communications network.

42. The system as claimed in claim 36, wherein the processor is used to carry out routing in a communications network. - -

**IN THE ABSTRACT**

Cancel the Abstract as filed, and insert therefore on a separate page, the following Abstract of the disclosure:

**-- ABSTRACT OF THE DISCLOSURE**

A determination of a sequence of actions is performed such that a sequence of states resulting from the sequence of actions is optimized using a prescribed optimization function. The optimization function includes a variable parameter with which it is possible to establish a risk relating to the resulting sequence of states based upon a prescribed state of the system. - -

**REMARKS**

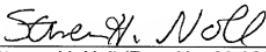
A substitute specification and a proper abstract of the disclosure are provided herewith which make editorial changes in order to conform to standard US practice. A marked-up copy of the specification is also provided reflecting the changes made.

In addition, the claims as filed have been canceled and replaced by new claims that more clearly set forth the subject matter of Applicants' invention.

No new matter has been inserted into the application.

Applicants submit that this application is in proper condition for examination in the United States National Stage, which action is earnestly solicited.

Respectfully submitted,

  
Steven H. Noll  
(Reg. No. 28,982)

SCHIFF HARDIN & WAITE  
Patent Department  
6600 Sears Tower  
233 South Wacker Drive  
Chicago, IL 60606  
Telephone: (312) 258-5790  
Attorneys for Applicant(s)

Customer Number: 26574

BOX PCT

IN THE UNITED STATES DESIGNATED/ELECTED OFFICE  
OF THE UNITED STATES PATENT AND TRADEMARK OFFICE  
UNDER THE PATENT COOPERATION TREATY – CHAPTER II

**SUBMISSION OF DRAWINGS**

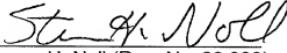
APPLICANT(S): NEUNEIER, R., et al.  
ATTORNEY DOCKET NO: P01,0020  
INTERNATIONAL APPLICATION NO: PCT/DE99/02846  
INTERNATIONAL FILING DATE: 8 SEP 1999  
INVENTION: METHOD AND ARRANGEMENT FOR  
DETERMINING A SEQUENCE OF  
ACTIONS FOR A SYSTEM

Assistant Commissioner for Patents  
Washington, DC 20231

Sir:

Applicants herewith submit four drawing sheets, showing Figures 1 – 6, in  
the captioned PCT application.

Respectfully submitted,

  
Steven H. Noll (Reg. No. 28,982)

SCHIFF, HARDIN & WAITE  
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6600 Sears Tower  
233 South Wacker Drive  
Chicago, IL 60606  
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Attorneys for Applicant(s)

Customer Number: 26574

FIG 1

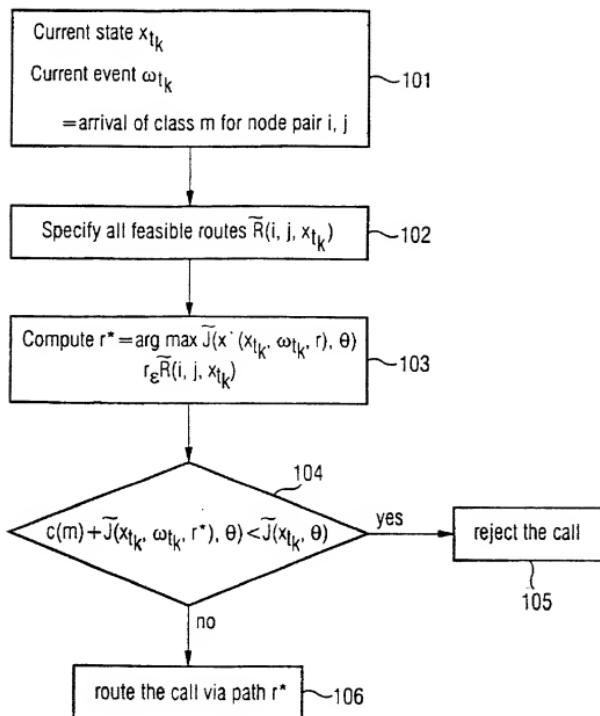


FIG 2

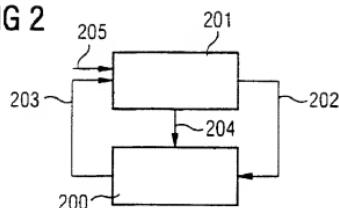


FIG3

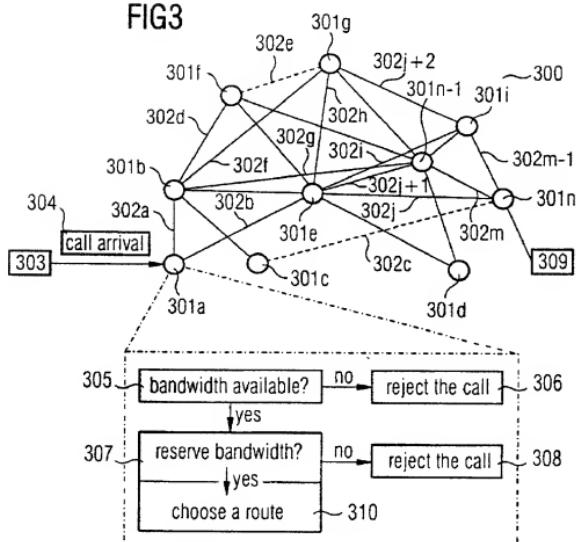


FIG 4

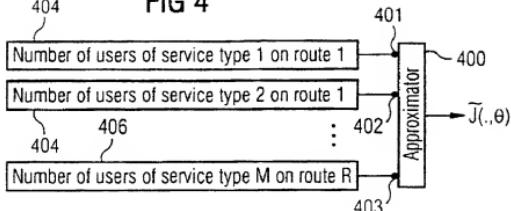


FIG 5

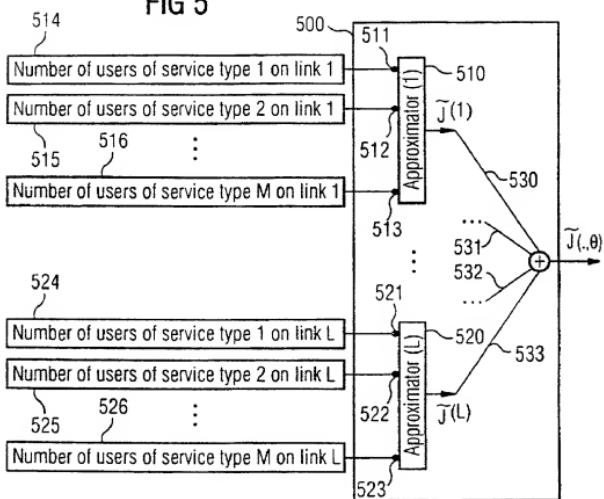
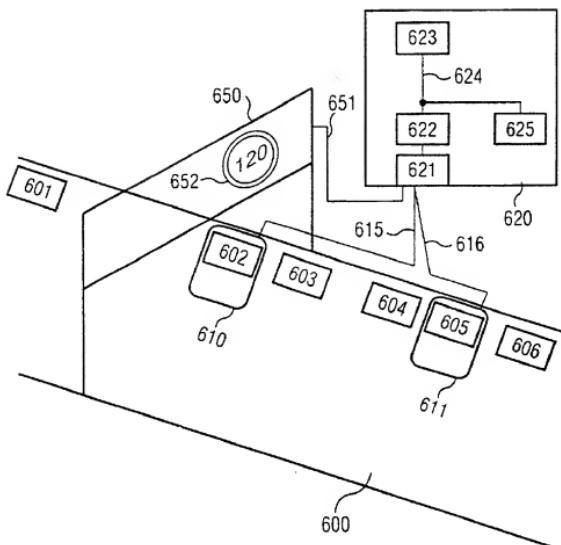


FIG 6



T012509-B63787698

Substitute specification:

**-- METHOD AND ARRANGEMENT FOR DETERMINING  
A SEQUENCE OF ACTIONS FOR A SYSTEM**

**BACKGROUND OF THE INVENTION**

**Field of the Invention:**

This invention generally pertains to systems having states, and in particular to methods for determining a sequence of actions for such systems.

**Discussion of the Related Art:**

A generalized method and arrangement for determining a sequence of actions for a system having states, wherein a transition in state between two states is performed on the basis of an action, is discussed by Neuneier in "Enhancing Q-Learning for Optimal Asset Allocation", appearing in the Proceedings of the Neural Information Processing Systems, NIPS 1997. Neuneier describes a financial market as an example of a system which has states. His system is described as a Markov Decision Problem (MDP).

The characteristics of a Markov Decision Problem are represented below by way of summary:

X	set of possible states of the system, e.g. $X = \mathbb{R}^m$ ,
$A(x_i)$	set of possible actions in the state

$$p(x_{t+1} | x_t, a_t)$$

$x_t$

$$r(x_t, a_t, x_{t+1})$$

gain with expectation  $R(x_t, a_t)$ .

Starting from observable variables, the variables denoted below as training data, the aim is to determine a strategy, that is to say a sequence of functions

$$\pi = \{\mu_0, \mu_1, \dots, \mu_T\}, \quad (3)$$

which at each instant  $t$  map each state into an action rule, that is to say action

$$\mu_t(x_t) = a_t \quad (4)$$

Such a strategy is evaluated by an optimization function.

The optimization function specifies the expectation, the gains accumulated over time at a given strategy  $\pi$ , and a start state  $x_0$ .

The so-called Q-learning method is described by Neuneier as an example of a method of approximative dynamic programming.

An optimum evaluation function  $V^*(x)$  is defined by

$$V^*(x) = \max_{\pi} V^{\pi}(x) \quad \forall x \in X \quad (5)$$

where

$$V^{\pi}(x) = E \left[ \sum_{t=0}^{\infty} \gamma^t r(x_t, \mu_t, x_{t+1}) | x_0 = x \right], \quad (6)$$

$\gamma$  denoting a prescribable reduction factor which is formed in accordance with the following rule:

$$\gamma = \frac{1}{1+z}, \quad (7)$$

$$z \in \mathbb{R}^+. \quad (8)$$

A Q-evaluation function  $Q^*(x_t, a_t)$  is formed within the Q-learning method for each pair (state  $x_t$ , action  $a_t$ ) in accordance with the following rule:

$$\begin{aligned} Q^*(x_t, a_t) = & \sum_{x \in X} p(x_{t+1}|x_t, a_t) \cdot r_t + \\ & + \gamma \cdot \sum_{x \in X} p(x|x_t, a_t) \cdot \max_{a \in A}(Q^*(x, a)) \end{aligned} \quad (9)$$

On the basis respectively of the tupel  $(x_t, x_{t+1}, a_t, r_t)$ , the Q-values  $Q^*(x, a)$  are adapted in the  $k+1$  th iteration in accordance with the following learning rule with a prescribed learning rate  $\eta_k$  in accordance with the following rule:

$$Q_{k+1}(x_t, a_t) = (1 - \eta_k)Q_k(x_t, a_t) + \eta_k \left( r_t + \gamma \max_{a \in A}(Q_k(x_{t+1}, a)) \right). \quad (10)$$

Usually, the so-called Q-values  $Q^*(x, a)$  are approximated for various actions by a function approximator in each case, for example a neural network or a polynomial classifier, with a weighting vector  $w^a$ , which contains weights of the function approximator.

A function approximator is, for example, a neural network, a polynomial classifier or a combination of a neural network with a polynomial classifier.

It therefore holds that:

$$Q^*(x, a) \approx Q(x; w^a). \quad (11)$$

Changes in the weights in the weighting vector  $w^a$  are based on a temporal difference  $d_t$  which is formed in accordance with the following rule:

$$d_t := r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}; w_k^a) - Q(x_t; w_k^{a_t}) \quad (12)$$

The following adaptation rule for the weights of the neural network, which are included in the weighting vector  $w^a$ , follows for the Q-learning method with the use of a neural network:

$$w_{k+1}^{a_t} = w_k^{a_t} + \eta_k \cdot d_t \cdot \nabla Q(x_t; w_k^{a_t}). \quad (13)$$

The neural network representing the system of a financial market as described by Neuneier is trained using the training data which describe information on changes in prices on a financial market as time series values.

A further method of approximative dynamic programming is the so-called TD( $\lambda$ ) learning method. This method is discussed in R.S. Sutton's, "Learning To Predict By The Method Of Temporal Differences", appearing in Machine Learning, Chapter 3, pages 9 - 44, 1988.

Furthermore, it is known from M. Heger's, "Risk and Reinforcement Learning":

Concepts and Dynamic Programming", ZKW Bericht No. 8/94, Zentrum für Kognitionswissenschaften [Center for Cognitive Sciences], Bremen University, December 1994, that risk is associated with a strategy  $\pi$  and an initial state  $x_t$ . A method for risk avoidance is also discussed by Hager, cited above.

The following optimization function, which is also referred to as an expanded Q-function  $\underline{Q}^*(x_t, a_t)$ , is used in the Hager method:

maximize

$$\underline{Q}^*(x_t, a_t) = r(x_t, a_t, x_{t+1}) + \inf_{\substack{x_0, x_1, K \\ p(x_0, x_1, K) > 0}} \left\{ \sum_{k=1}^{\infty} \gamma^k r(x_k, \pi(x_k), x_{k+1}) \right\} \quad (14)$$

The expanded Q-function  $\underline{Q}^*(x_t, a_t)$  describes the worst case if the action  $a_t$  is executed in the state  $x_t$  and the strategy  $\pi$  is followed thereupon.

The optimization function  $\underline{Q}^*(x_t, a_t)$  for

$$\underline{Q}^*(x_t, a_t) := \max_{\pi \in \Pi} \underline{Q}^*(x_t, a_t) \quad (15)$$

is given by the following rule:

$$\underline{Q}^*(x_t, a_t) = \min_{\substack{x \in X \\ p(x_{t+1}|x_t, a_t) > 0}} \left( r(x_t, a_t, x) + \gamma \cdot \max_{a \in A} \underline{Q}^*(x, a) \right). \quad (16)$$

A substantial disadvantage of this mode of procedure is that only the worst case is taken into account when finding the strategy. However, this inadequately reflects the requirements of the most varied technical systems.

In "Dynamic Programming and Optimal Control", Athena Scientific, Belmont, MA, 1995, D.P. Bertsekas formulates access control for a communications network and routing within the communications network as a problem of dynamic programming.

Therefore, the present invention is based on the problem of specifying a method and system for determining a sequence of actions in which the method or sequences of actions achieve an increased flexibility in determining the strategy needed.

In a method for computer-aided determination of a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, the determination of the sequence of actions is performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization function including a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

A system for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, has a processor which is set up in such a way that the determination of the

sequence of actions can be performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization function including a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

Thus, the present invention offers a method for determining a sequence of actions at a freely prescribable level of accuracy when finding a strategy for a possible closed-loop control or open-loop control of the system, in general for influencing it. Hence, the embodiments described below are valid both for the method and for the system.

Approximative dynamic programming is used for the purpose of determination, for example a method based on Q-learning or a method based on TD( $\lambda$ )-learning.

Within Q-learning, the optimization function OFQ is preferably formed in accordance with the following rule:

$$OFQ = Q(x; w^a),$$

- $x$  denoting a state in a state space  $X$
- $a$  denoting an action from an action space  $A$ , and
- $w^a$  denoting the weights of a function approximator which belong to the action  $a$ .

The following adaptation step is executed during Q-learning in order to determine the optimum weights  $w^a$  of the function approximator:

$$w_{t+1}^{a_t} = w_t^{a_t} + \eta_t \cdot \kappa^\kappa(d_t) \cdot \nabla Q(x_t; w_t^{a_t})$$

with the abbreviation

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{a_t})$$

- $x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,
- $a_t$  denoting an action from an action space  $A$ ,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t^{a_t}$  denoting the weighting vector associated with the action  $a_t$  before the adaptation step,
- $w_{t+1}^{a_t}$  denoting the weighing vector associated with the action  $a_t$  after the adaptation step,
- $\eta_t (t = 1, \dots)$  denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk monitoring parameter,
- $\kappa^\kappa$  denoting a risk monitoring function  $\kappa^\kappa(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- $\nabla Q(\cdot)$  denoting the derivation of the function approximator according to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

The optimization function is preferably formed in accordance with the following

rule within the TD( $\lambda$ )-learning method:

$$\text{OFTD} = J(x; w)$$

- $x$  denoting a state in a state space  $X$ ,
- $a$  denoting an action from an action space  $A$ , and
- $w$  denoting the weights of a function approximator.

The following adaptation step is executed during TD( $\lambda$ )-learning in order to determine the optimum weights  $w$  of the function approximator:

$$w_{t+1} = w_t + \eta_t \cdot N^\kappa(d_t) \cdot z_t$$

with the abbreviations

$$d_t = r(w_t, a_t, x_{t+1}) + \gamma J(x_{t+1}; w_t) - J(x_t; w_t),$$

$$z_t = \lambda \cdot \gamma \cdot z_{t-1} + \nabla J(x_t; w_t),$$

$$z_1 = 0$$

- $x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,
- $a_t$  denoting an action from an action space  $A$ ,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t$  denoting the weighting vector before the adaptation step,
- $w_{t+1}$  denoting the weighting vector after the adaptation step,
- $\eta_t (t = 1, \dots)$  denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk monitoring parameter,
- $N^\kappa$  denoting a risk monitoring function  $N^\kappa(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- $\nabla J(\cdot)$  denoting the derivation of the function approximator according to its

weights, and

- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

### SUMMARY OF THE INVENTION

It is an object of the present invention to provide a technical system and method for determining a sequence of actions using measured values.

It is another object of the present invention to provide a technical system and method that can be subjected to open-loop control or closed-loop control with the use of a determined sequence of actions.

It is a further object of the invention to provide a technical system and method modeled as a Markov Decision Problem.

It is an additional object of the invention to provide a technical system and method that can be used in a traffic management system.

It is yet another object of the invention to provide a technical system and method that can be used in a communications system, such that a sequence of actions is used to carry out access control, routing or path allocation.

It is yet a further object of the invention to provide a technical system and

method for a financial market modeled by a Markov Decision Problem, wherein a change in an index of stocks, or a change in a rate of exchange on a foreign exchange market, makes it possible to intervene in the market in accordance with a sequence of determined actions.

These and other objects of the invention will be apparent from a careful review of the following detailed description of the preferred embodiments, which is to read in conjunction with a review of the accompanying drawing figures.

#### **BRIEF DESCRIPTION OF THE DRAWINGS**

- Figure 1 shows a flowchart of method steps according to the present invention;
- Figure 2 shows a system modeled as a Markov Decision Problem;
- Figure 3 shows a communications network wherein access control is carried out in a switching unit according to the present invention;
- Figure 4 shows a function approximator for approximative dynamic programming according to the present invention;
- Figure 5 shows a plurality of function approximators for approximative dynamic programming according to the present invention; and
- Figure 6 shows a traffic management system subjected to closed-loop control in accordance with the present invention.

#### **DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS**

Figure 1 shows a flowchart according to the present invention, in which

individual method steps of a first embodiment are provided, which will be discussed later.

Figure 2 shows the structure of a typical Markov Decision Problem method.

The system 201 is in a state  $x_t$  at an instant  $t$ . The state  $x_t$  can be observed by an observer of the system. On the basis of an action  $a_t$  from a set in the state  $x_t$  of possible actions,  $a_t \in A(x_t)$ , the system makes a transition with a certain probability into a subsequent state  $x_{t+1}$  at a subsequent instant  $t+1$ .

As illustrated diagrammatically in Figure 2 by a loop, an observer 200 perceives 202 observable variables concerning the state  $x_t$  and takes a decision via an action 203 with which it acts on the system 201. The system 201 is usually subject to the interference 205.

The observer 200 obtains a gain  $r_t$  204

$$r_t = r(x_t, a_t, x_{t+1}) \in \mathbb{R}, \quad (1)$$

which is a function of the action  $a_t$  203 and the original state  $x_t$  at the instant  $t$  as well as of the subsequent state  $x_{t+1}$  of the system at the subsequent instant  $t+1$ .

The gain  $r_t$  can assume a positive or negative scalar value depending on whether the decision leads, with regard to a prescribable criterion, to a positive or negative system development, to an increase in capital stock or to a loss.

In a further time step, the observer 200 of the system 201 decides on the basis of the observable variables 202, 204 of the subsequent state  $x_{t+1}$  in favor of a new action  $a_{t+1}$ , etc.

A sequence of

State:  $x_t \in X$   
Action:  $a_t \in A(x_t)$   
Subsequent state:  $x_{t+1} \in X$   
Gain  $r_t = r(x_t, a_t, x_{t+1}) \in \mathcal{R}$

describes a trajectory of the system which is evaluated by a performance criterion which accumulates the individual gains  $r_t$  over the instants  $t$ . It is assumed by way of simplification in a Markov Decision Problem that the state  $x_t$  and the action  $a_t$  all contain information for the purpose of describing a transition probability  $p(x_{t+1} | \cdot)$  of the system from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

In formal terms, this means that:

$$p(x_{t+1} | x_t, K, x_0, a_t, K, a_0) = p(x_{t+1} | x_t, a_t). \quad (2)$$

$p(x_{t+1} | x_t, a_t)$  denotes a transition probability for the subsequent state  $x_{t+1}$  for a given state  $x_t$  and given action  $a_t$ .

In a Markov Decision Problem, future states of the system 201 are thus not a function of states and actions which lie further in the past than one time step.

Figure 3 shows an embodiment of the present invention involving an access control and routing system, such as a communications network 300.

The communications network 300 has a multiplicity of switching units 301a, 301b, ..., 301i, ... 301n, which are interconnected via connections 302a, 302b, 302j, ... 302m. A first terminal 303 is connected to a first switching unit 301a. From the first terminal 303, the first switching unit 301a is sent a request message 304 which requests preservation of a prescribed bandwidth within the communications network 300 for the purpose of transmitting data, such as video data or text data.

It is determined in the first switching unit 301a in accordance with a strategy described below, whether the requested bandwidth is available in the communications network 300 on a specified, requested connection instep 305. The request is refused instep 306 if this is not the case. If sufficient bandwidth is available, it is checked in checking step 307 whether the bandwidth can be reserved.

The request is refused in step 308 if this is not the case. Otherwise, the first switching unit 301a selects a route from the first switching unit 301a via further switching units 301i to a second terminal 309 with which the first terminal 303 wishes to communicate, and a connection is initialized in step 310.

The starting point below is a communications network 300 which comprises a set of switching units

$$N = \{1, K_1, n, K_n, N\} \quad (17)$$

and a set of physical connections

$$L = \{1, K_1, 1, K_2, L\}, \quad (18)$$

a physical connection  $I$  having a capacity of  $B(I)$  bandwidth units.

A set

$$M = \{1, K_1, m, K_2, M\} \quad (19)$$

of different types of service  $m$  are available, a type of service  $m$  being characterized by

- a bandwidth requirement  $b(m)$ ,
- an average connection time  $\frac{1}{v(m)}$ , and
- a gain  $c(m)$  which is obtained whenever a call request of the corresponding type of service  $m$  is accepted.

The gain  $c(m)$  is given by the amount of money which a network operator of the communications network 300 bills a subscriber for a connection of the type of service. Clearly, the gain  $c(m)$  reflects different priorities, which can be prescribed by the network operator and which he associates with different services.

A physical connection  $1$  can simultaneously provide any desired combination of communications connections as long as the bandwidth used for the communications connections does not exceed the bandwidth available overall for the physical connection.

If a new communications connection of type  $m$  is requested between a first node  $i$  and a second node  $j$  (terminals are also denoted as nodes), the requested

communications connection can, as represented above, either be accepted or be refused. If the communications connection is accepted, a route is selected from a set of prescribed routes. This selection is denoted as a routing.  $b(m)$  bandwidth units are used in the communications connection of type  $m$  for each physical connection along the selected route for the duration of the connection.

Thus, during access control, also referred to as call admission control, a route can be selected within the communications network 300 only when the selected route has sufficient bandwidth available. The aim of the access control and of the routing is to maximize a long term gain which is obtained by acceptance of the requested connections.

At an instant  $t$ , the technical system which is the communications network 300 is in a state  $x_t$  which is described by a list of routes via existing connections, by means of which lists it is shown how many connections of which type of service are using the respective routes at the instant  $t$ .

Events  $w$ , by means of which a state  $x_t$  could be transferred into a subsequent state  $x_{t+1}$ , are the arrival of new connection request messages, or else the termination of a connection existing in the communications network 300.

In this embodiment, an action  $a_t$  at an instant  $t$ , owing to a connection request is the decision as to whether a connection request is to be accepted or refused and, if the connection is accepted, the selection of the route through the communications network 300.

The aim is to determine a sequence of actions, that is to say clearly to determine the learning of a strategy with actions relating to a state  $x_t$  in such a way that the following rule is maximized:

$$E \left( \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}) \right), \quad (20)$$

- $E(\cdot)$  denoting an expectation,
- $t_k$  denoting an instant at which a  $k$ th event takes place,
- $g(x_{t_k}, \omega_k, a_{t_k})$ . denoting the gain which is associated with the  $k$ th event, and
- $\beta$  denoting a reduction factor which evaluates an immediate gain as being more valuable than a gain at instants lying further in the future.

Different implementations of a strategy lead normally to different overall gains

G:

$$G = \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}). \quad (21)$$

The aim is to maximize the expectation of the overall gain G in accordance with the following rule J:

$$J = E \left\{ \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}) \right\}, \quad (22)$$

it being possible to set a risk which reduces the overall gain G of a specific implementation of access control and of a routing strategy to below the expectation.

The TD( $\lambda$ )-learning method is used to carry out the access control and the

routing.

The following target function is used in this embodiment:

$$J^*(x_t) = \mathbb{E}_\tau \left\{ e^{-\beta \tau} \right\} \mathbb{E}_\omega \left\{ \max_{a \in A} [g(x_t, \omega_t, a) + J^*(x_{t+1})] \right\}, \quad (23)$$

- $A$  denoting an action space with a prescribed number of actions which are respectively available in a state  $x_t$ ,
- $\tau$  denoting a first instant at which a first event  $\omega$  occurs, and
- $x_{t+1}$  denoting a subsequent state of the system.

An approximated value of the target value  $J^*(x_t)$  is learned and stored by employing a function approximator 400 (compare Figure 4) with the use of training data.

Training data are data previously measured in the communications network 300 and relating to the behavior of the communications network 300 in the case of incoming connection requests 304 and of termination of messages. This time sequence of states is stored, and these training data are used to train the function approximator 400 in accordance with the learning method described below.

A number of connections of in each case one type of service  $m$  on a route of the communications network 300 serve in each case as input variable of the function approximator 400 for each input 401, 402, 403 of the function approximator 400. These are represented in Figure 4 by blocks 404, 405, 406. An approximated target value  $\tilde{J}$  of the target value  $J^*$  is the output variable of the function approximator 400.

Figure 5 shows a detailed representation of a function approximator 500, which has several component function approximators 510, 520.

One output variable is the approximated target value  $\tilde{J}$ , which is formed in accordance with the following rule:

$$\tilde{J}(x_t, \Theta) = \sum_{l=1}^L \tilde{J}^{(l)}(x_t^{(l)}, \Theta_t^{(l)}). \quad (24)$$

The input variables of the component function approximators 510, 520, which are present at the inputs 511, 512, 513 of the first component function approximator 510, or at the inputs 521, 522 and 523 of the second component function approximator 520 are, in turn, respectively a number of types of service of a type m in a physical connection r in each case, symbolized by blocks 514, 515, 516 for the first component function approximator, and 524, 525 and 526 for the second component function approximator 520.

Component output variables 530, 531, 532, 533 are fed to an adder unit 540, and the approximated target variable  $\tilde{J}$  is formed as output variable of the adder unit.

Let it be assumed that the communications network 300 is in the state  $x_{tk}$  and that a request message with which a type of service m of class m is requested for a connection between two nodes i, j reaches the first switching unit 301a.

A list of permitted routes between the nodes  $i$  and  $j$  is denoted by  $R(i, j)$ , and a list of all possible routes is denoted by

$$\tilde{R}(i, j, x_{t_k}) \subset R(i, j) \quad (25)$$

as a subset of the routes  $R(i, j)$  which could implement a possible connection with regard to the available and requested bandwidth.

For each possible route  $r$ ,  $r \in \tilde{R}(i, j, x_{t_k})$ , a subsequent state

$x_{t_k+1}(x_{t_k}, \omega_k, r)$  is determined which results from the fact that the connection request 304 is accepted and the connection on the route  $r$  is made available to the requesting first terminal 303.

This is illustrated in Figure 1 as step 102, the state of the system and the respective event being respectively determined in step 101. A route  $r^*$  to be selected is determined in step 103 in accordance with the following rule:

$$r^* = \arg \max_{r \in \tilde{R}(i, j, x_{t_k})} \tilde{j}(x_{t_k+1}(x_{t_k}, \omega_k, r), \Theta_t). \quad (26)$$

A check is made in step 104 as to whether the following rule is fulfilled:

$$c(m) + \tilde{j}(x_{t_k+1}(x_{t_k}, \omega_k, r^*), \Theta_t) < \tilde{j}(x_{t_k}, \Theta_t). \quad (27)$$

If this is the case, the connection request 304 is rejected in step 105, otherwise the connection is accepted and "switched through" to the node  $j$  along the selected route  $r^*$  in step 106.

Weights of the function approximator 400, 500 which are adapted in the TD( $\lambda$ )-learning method to the training data, are stored in a parameter vector  $\theta$  for an instant  $t$  in each case, such that an optimized access control and an optimized routing are achieved.

During the training phase, the weighting parameters are adapted to the training data applied to the function approximator.

A risk parameter  $\kappa$  is defined with the aid of which a desired risk, which the system has with regard to a prescribed state owing to a sequence of actions and states, can be set in accordance with the following rules:

- 1  $\leq \kappa < 0$ : risky learning,
- $\kappa = 0$ : neutral learning with regard to the risk,
- $0 < \kappa < 1$ : risk-avoiding learning,
- $\kappa = 1$ : worst-case learning.

Furthermore, a prescribable parameter  $0 \leq \lambda \leq 1$  and a step size sequence  $\gamma_k$  are prescribed in the learning method.

The weighting values of the weighting vector  $\Theta$  are adapted to the training data on the basis of each event  $\omega_{tk}$  in accordance with the following adaptation rule:

$$\Theta_k = \Theta_{k-1} + \gamma_k N^\kappa(d_k) z_t, \quad (28)$$

in which case

$$d_k = e^{-\beta(t_k - t_{k-1})} (g(x_{t_k}, \omega_k, a_{t_k}) + \tilde{J}(x_{t_k}, \Theta_{k-1})) - \tilde{J}(x_{t_{k-1}}, \Theta_{k-1}) \quad (29)$$

$$z_t = \lambda e^{-\beta(t_{k-1} - t_{k-2})} z_{t-1} + \nabla_{\Theta} \tilde{J}(x_{t_{k-1}}, \Theta_{k-1}), \quad (30)$$

and

$$\kappa^K(\xi) = (1 - \kappa \text{sign}(\xi)) \xi. \quad (31)$$

It is assumed that:  $z_1 = 0$ .

The function

$$g(x_{t_k}, \omega_k, a_{t_k}) \quad (32)$$

denotes the immediate gain in accordance with the following rule:

$$g(x_{t_k}, \omega_k, a_{t_k}) = \begin{cases} c(m) & \text{when } \omega_{t_k} \text{ is a service request for a type of} \\ & \text{service } m, \text{ and the connection is accepted} \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

Thus, as described above, a sequence of actions is determined with regard to a connection request such that a connection request is either rejected or accepted on the basis of an action. The determination is performed taking account of an optimization function in which the risk can be set by means of a risk control parameter  $\kappa \in [-1; 1]$  in a variable fashion.

Figure 6 shows an embodiment of the present invention in relation to a traffic management system

A road 600 on which automobiles 601, 602, 603, 604, 605 and 606 are being driven. Conductor loops 610, 611 integrated into the road 600 receive electric signals in a known way and feed the electric signals 615, 616 to a computer 620 via an input/output interface 621. In an analog-to-digital converter 622 connected to the input/output interface 621, the electric signals are digitized into a time series and stored in a memory 623, which is connected by a bus 624 to the analog-to-digital converter 622 and a processor 625. Via the input/output interface 621, a traffic management system 650 is fed control signals 651 from which it is possible to set a prescribed speed stipulation 652 in the traffic management system 650, or else further particulars of traffic regulations, which are displayed via the traffic management system 650 to drivers of the vehicles 601, 602, 603, 604, 605 and 606.

The following local state variables are used in this case for the purpose of traffic modeling:

- traffic flow rate  $v$ ,
- vehicle density  $\rho$  ( $\rho$  = number of vehicles per kilometer  $\frac{Fz}{km}$ ),
- traffic flow  $q$  ( $q$  = number of vehicles per hour  $\frac{Fz}{h}$ , ( $q=v * \rho$ )), and
- speed restrictions 652 displayed by the traffic management system 650 at an instant in each case.

The local state variables are measured as described above by using the conductor loops 610, 611.

These variables ( $v(t)$ ,  $\rho(t)$ ,  $q(t)$ ) therefore represent a state of the technical system of "traffic" at a specific instant  $t$ .

In this embodiment, the system is therefore a traffic system which is controlled by using the traffic management system 650, and an extended Q-learning method is described as method of approximative dynamic programming.

The state  $x_t$  is described by a state vector  
$$x(t) = (v(t), \rho(t), q(t)). \quad (34)$$

The action  $a_t$  denotes the speed restriction 652, which is displayed at the instant  $t$  by the traffic management system 650. The gain  $r(x_t, a_t, x_{t+1})$  describes the quality of the traffic flow which was measured between the instants  $t$  and  $t+1$  by the conductor loops 610 and 611.

In this embodiment,  $r(x_t, a_t, x_{t+1})$  denotes

- the average speed of the vehicles in the time interval  $[t, t + 1]$

or

- the number of vehicles which have passed the conductor loops 610 and 611 in the time interval  $[t, t + 1]$

or

- the variance of the vehicle speeds in the time interval  $[t, t + 1]$ ,

or

- a weighted sum from the above variables.

A value of the optimization function OFQ is determined for each possible action  $a_t$ , that is to say for each speed restriction which can be displayed by the traffic management system 650, an estimated value of the optimization function OFQ being realized in each case as a neural network.

This results in a set of evaluation variables for the various actions  $a_t$  in the system state  $x_t$ . Those actions  $a_t$  for which the maximum evaluation variable OFQ has been determined in the current system state  $x_t$  are selected in a control phase from the possible actions  $a_t$ , that is to say from the set of the speed restrictions which can be displayed by the traffic management system 650.

In accordance with this embodiment, the adaptation rule, known from the Q-learning method, for calculating the optimization function OFQ is extended by a risk control function  $\kappa(\cdot)$ , which takes account of the risk.

In turn, the risk control parameter  $\kappa$  is prescribed in accordance with the strategy from the first exemplary embodiment in the interval of  $[-1 \leq \kappa \leq 1]$ , and represents the risk which a user wishes to run in the application with regard to the control strategy to be determined.

The following evaluation function OFQ is used in accordance with this exemplary embodiment:

$$OFQ = Q(x; w^a), \quad (35)$$

- $x = (v; p; q)$  denoting a state of the traffic system,
- $a$  denoting a speed restriction from the action space A of all speed restrictions which can be displayed by the traffic management system 650, and
- $w^a$  denoting the weights of the neural network which belong to the speed restriction a.

The following adaptation step is executed in Q-learning in order to determine the optimum weights  $w^a$  of the neural network:

$$w_{t+1}^{at} = w_t^{at} + \eta_t \cdot N^K(d_t) \cdot \nabla Q(x_t; w_t^{at}) \quad (36)$$

using the abbreviation:

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{at}) \quad (37)$$

- $x_t, x_{t+1}$  denoting in each case a state of the traffic system in accordance with rule (34),
- $a_t$  denoting an action, that is to say a speed restriction which can be displayed by the traffic management system 650,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t^{at}$  denoting the weighting vector belonging to the action  $a_t$ , before the adaptation step,

- $w_{t+1}^{a_t}$  denoting the weighting vector belonging to the action  $a_t$ , after the adaptation step,
- $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk control parameter,
- $N^{\kappa}$  denoting a risk control function  $N^{\kappa}(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- $\nabla_{\theta} f(\cdot)$  denoting the derivative of the neural network with respect to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition in state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

An action  $a_t$  can be selected at random from the possible actions  $a_t$  during learning. It is not necessary in this case to select the action  $a_t$  which has led to the largest evaluation variable.

The adaptation of the weights has to be performed in such a way that not only is a traffic control achieved which is optimized in terms of the expectation of the optimization function, but that also account is taken of a variance of the control results.

This is particularly advantageous since the state vector  $x(t)$  models the actual system of traffic only inadequately in some aspects, and so unexpected disturbances can thereby occur. Thus, the dynamics of the traffic, and therefore of its modeling, depend on further factors such as weather, proportion of trucks on the road, proportion of mobile homes, etc., which are not always integrated in the measured

variables of the state vector  $x(t)$ . In addition, it is not always ensured that the road users immediately implement the new speed instructions in accordance with the traffic management system.

A control phase on the real system in accordance with the traffic management system takes place in accordance with the following steps:

1. The state  $x_t$  is measured at the instant  $t$  at various points in the traffic system of traffic and yields a state vector  $x(t) = (v(t), p(t), q(t))$ .
2. A value of the optimization function is determined for all possible actions  $a_t$ , and that action  $a_t$  with the highest evaluation in the optimization function is selected.

Although modifications and changes may be suggested by those skilled in the art to which this invention pertains, it is the intention of the inventors to embody within the patent warranted hereon all changes and modifications that may reasonably and properly come under the scope of their contribution to the art. --

{Description} [Substitute specification:]

{Method and arrangement}[- - METHOD AND ARRANGEMENT FOR DETERMINING  
A SEQUENCE OF ACTIONS FOR A SYSTEMBACKGROUND OF THE INVENTION

## Field of the Invention:

This invention generally pertains to systems having states, and in particular to methods] for determining a sequence of actions for {a system which has states,}[such systems.

## Discussion of the Related Art:

A generalized method and arrangement for determining a sequence of actions for a system having states, wherein] a transition in state between two states {being} [is] performed on the basis of an action[, is discussed by Neuneier in \*Enhancing Q-Learning for Optimal Asset Allocation\*, appearing in the Proceedings of the Neural Information Processing Systems, NIPS 1997. Neuneier describes a financial market as an example of] {The invention relates to a method and an arrangement for determining a sequence of actions for} a system which has states[. His]{-a transition in state between two states being performed on the basis of an action.

Such a method and such an arrangement are known from [1].

A financial market is described in [1] as an example for such a system which has states.

The} system is described as a Markov [Decision Problem (MDP).] {decision problem (MDP).} The structure of a system which can be described as a Markov decision problem is illustrated in Figure 2.

The system 201 is in a state  $x_t$  at an instant  $t$ . The state  $x_t$  can be observed by an observer of the system. On the basis of an action  $a_t$  from a set in the state  $x_t$  of possible actions, at {- $A(x_t)$ }, the system makes a transition with a certain probability into a subsequent state  $x_{t+1}$  at a subsequent instant  $t+1$ .

This is illustrated diagrammatically in Figure 2 by a loop. An observer 200 perceives 202 observable variables concerning the state  $x_t$  and takes a decision via an action 203 with which it acts on the system 201. The system 201 is usually subject to the interference 205.

Furthermore, the observer 200 obtains a gain  $r_t$  204

which is a function of the action  $a_t$  203 and the original state  $x_t$  at the instant  $t$  as well as of the subsequent state  $x_{t+1}$  of the system at the subsequent instant  $t+1$ .

The gain  $r_t$  can assume a positive or negative scalar value, depending on whether the decision leads, with regard to a prescribable criterion, to a positive or negative system development, in [1] to an increase in capital stock or to a loss.

In a further time step, the observer 200 of the system 201 decides on the basis of the observable variables 202, 204 of the subsequent state  $x_{t+1}$  in favor of a new action  $a_{t+1}$ , etc.

A sequence of

State:  $x_t$  - X

Action:  $a_t$  - A( $x_t$ )

Subsequent state:  $x_{t+1}$  - X

Gain:  $r_t = r(x_t, a_t, x_{t+1})$  {()

etc. describes a trajectory of the system which is evaluated by a performance criterion which accumulates the individual gains  $r_t$  over the instants  $t$ . It is assumed by way of simplification in a Markov decision problem that the state  $x_t$  and the action  $a_t$  all contain information for the purpose of describing a transition probability  $p(x_{t+1}|x_t)$  of the system from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

In formal terms, this means that:

$p(x_{t+1}|x_t, a_t)$  denotes a transition probability for the subsequent state  $x_{t+1}$  for a given state  $x_t$  and given action  $a_t$ .

In a Markov decision problem, future states of the system 201 are thus not a function of states and actions which lie further in the past than one time step.}

The characteristics of a Markov (decision problem) [Decision Problem] are represented below by way of summary:

X

set of possible states of the system,

e.g.  $X = \mathbb{R}^{10^m}$ ,

$A(x_t)$

set of possible actions in the state

$\{p(x_{t+1}|x_t) | p(x_{t+1}|x_t, a_t), a_t\}$

$x_t$

$r(x_t, a_t, x_{t+1})$  gain with expectation  $R(x_t, a_t)$ .

Starting from observable variables, the variables denoted below as training data, the aim is to determine a strategy, that is to say a sequence of functions

$$\pi = \{\mu_0, \mu_1, K, \mu_T\}, \quad (3)$$

which at each instant t map each state into an action rule, that is to say action

$$\mu_t(x_t) = a_t \quad (4)$$

Such a strategy is evaluated by an optimization function.

The optimization function specifies the expectation, the gains accumulated [ ] over time at a given strategy  $\pi(\cdot)$ , and a start state  $x_0$ .

The so-called Q-learning method is described [in-[1]] [by Neuneier] as an example of a method of approximative dynamic programming.

An optimum evaluation function  $V^*(x)$  is defined by

$$V^*(x) = \max_{\pi} V^{\pi}(x) \quad \forall x \in X \quad (5)$$

where

$$v^\pi(x) = E \left[ \sum_{t=0}^{\infty} \gamma^t r(x_t, \mu_t, x_{t+1}) | x_0 = x \right], \quad (6)$$

$\gamma$  {denoting a prescribable reduction factor which is formed in accordance with the following rule:

$$\gamma = \frac{1}{1+z}, \quad (7)$$

$$z \in \mathbb{R}^+. \quad (8)$$

A Q-evaluation function  $Q^*(x_t, a_t)$  is formed within the Q-learning method for each pair (state  $x_t$ , action  $a_t$ ) in accordance with the following rule:

$$Q^*(x_t, a_t) = \sum_{x \in X} p(x_{t+1} | x_t, a_t) \cdot r_t + \\ + \gamma \cdot \sum_{x \in X} p(x | x_t, a_t) \cdot \max_{a \in A} (Q^*(x, a)) \quad (9)$$

On the basis respectively of the tuple  $(x_t, x_{t+1}, a_t, r_t)$ , the Q-values  $Q^*(x, a)$  are [adapted in the  $k+1$  th iteration in accordance with the following learning rule with a prescribed learning rate  $\eta_{k+1}$  in accordance with the following rule:

$$Q_{k+1}(x_t, a_t) = (1 - \eta_k) Q_k(x_t, a_t) + \eta_k \left( r_t + \gamma \max_{a \in A} (Q_k(x_{t+1}, a)) \right). \quad (10)$$

Usually, the so-called Q-values  $Q^*(x, a)$  are approximated for various actions  $\{a\}$  by a function approximator in each case, for example a neural network or {else} a polynomial classifier, with a weighting vector  $w^a$ , which contains weights of the function approximator.

A function approximator is {to-be-understood-as}, for example, a neural network, a polynomial classifier or {else} a combination of a neural network with a polynomial classifier.

It therefore holds that:

$$Q^*(x, a) \approx Q(x; w^a). \quad (11)$$

Changes in the weights in the weighting vector  $w^a$  are based on a temporal difference  $d_t$  which is formed in accordance with the following rule:

$$d_t := r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}; w_k^a) - Q(x_t; w_k^{a_t}) \quad (12)$$

The following adaptation rule for the weights of the neural network, which are included in the weighting vector  $w^a$ , follows for the Q-learning method with the use of a neural network:

$$w_{k+1}^{a_t} = w_k^{a_t} + \eta_k \cdot d_t \cdot \nabla Q(x_t; w_k^{a_t}). \quad (13)$$

The neural network representing the system of a financial market [1] as described [in [1]] [by Neuneier] is trained using the training data which describe information on changes in prices on a financial market as time series values.

A further method of approximative dynamic programming, the so-called TD( $\lambda$ ) learning method, is known from [2] and is explained in more detail in conjunction with an exemplary embodiment. [is the so-called TD( $\lambda$ ) learning method. This method is discussed in R.S. Sutton's, "Learning To Predict By The Method Of Temporal Differences", appearing in Machine Learning, Chapter 3, pages 9 - 44, 1988.]

Furthermore, it is known from [[3]-which] [M. Heger's, "Risk and Reinforcement Learning: Concepts and Dynamic Programming", KZW Bericht No. 8/94, Zentrum für Kognitionswissenschaften [Center for Cognitive Sciences], Bremen University, December 1994, that] risk is associated with a strategy  $\pi$  and an initial [

]state  $x_t$ . A method for risk avoidance is likewise known from [3]) avoidance is also discussed by Hager, cited above].

The following optimization function, which is also referred to as an expanded Q-function ( $\underline{Q}^*(x)$ ) [ $\underline{Q}^*(x_t, a_t)$ , is used in the [Hager] method (known from [3]):

maximize

$$\left\{ \underline{Q}^*(x_t, a_t) = r(x_t, a_t, x_{t+1}) + \inf_{\substack{x_0, x_1, K \\ p(x_0, x_1, K) > 0}} \left\{ \sum_{k=1}^{\infty} \gamma^k r(x_k, \pi(x_k), x_{k+1}) \right\} \right\} \quad (14)$$

The expanded Q-function ( $\underline{Q}^*(x)$ ) [ $\underline{Q}^*(x_t, a_t)$  describes the worst case if the action  $a_t$  is executed in the state  $x_t$  and the strategy  $\pi(\cdot)$  is followed thereupon.

The optimization function ( $\underline{Q}^*(x)$ ) [ $\underline{Q}^*(x_t, a_t)$  for

$$\underline{Q}^*(x_t, a_t) := \max_{\pi \in \Pi} \underline{Q}^*(x_t, a_t) \quad (15)$$

is given by the following rule:

$$\underline{Q}^*(x_t, a_t) = \min_{\substack{x \in X \\ p(x_{t+1}|x_t, a_t) > 0}} \left( r(x_t, a_t, x) + \gamma \cdot \max_{a \in A} \underline{Q}^*(x, a) \right). \quad (16)$$

A substantial disadvantage of this mode of procedure is (to be seen in) that only the worst case is taken into account when finding the strategy. However, this [inadequately] reflects the requirements of the most varied technical systems (only to an inadequate extent).[]

{Furthermore, it is known from [4] to formulate} [In \*Dynamic Programming and Optimal Control\*, Athena Scientific, Belmont, MA, 1995, D.P. Bertsekas formulates] access control for a communications network [ ] and {the} routing within the communications network as a problem of dynamic [ ] programming.

{The} [Therefore, the present] invention is {therefore} based on the problem of specifying a method and {an arrangement} [system] for determining a sequence of actions {for a system} in which [the] method or {action} [sequences of actions achieve] an increased flexibility in determining the strategy {is achieved} [needed.]

{The problem is solved by the method and by the arrangement in accordance with the features of the independent patent claims.

In a method for computer-aided determination of a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, the determination of the sequence of actions is performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization function including a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

An arrangement for determining) [In a method for computer-aided determination of] a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, {has a processor which is set up in such a way that} the determination of the sequence of actions {can be} [is] performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization function including a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

[A system for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, has a processor which is set up in such a way that the determination of the sequence of actions can be performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization function including a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

Thus, the present invention offers] {It becomes possible for the first time owing to the invention to specify} a method for determining a sequence of actions at a freely prescribable level of accuracy when finding a strategy for a possible closed-loop control or open-loop control of the system, in general for influencing it. [Hence, the embodiments] {Preferred developments of the invention follow from the dependent claims.}

The developments} described below are valid both for the method and for the {arrangement, the processor being respectively set up in the development of arrangement in such a way that the development can be implemented;} [system.]

{In a preferred refinement, a method of approximative} [Approximative] dynamic programming is used for the purpose of {determination} [determination], for example a method based on Q-learning or {else} a method based on  $TD(\lambda)$ -learning.

Within Q-learning, the optimization function OFQ is preferably formed in accordance with the following rule:

$$OFQ = Q(x; w^a),$$

- {}x denoting a state in a state space X
- {}a denoting an action from an action space A, and
- {}w<sup>a</sup> denoting the weights of a function approximator which belong to the action a.

The following adaptation step is executed during Q-learning in order to determine the optimum weights  $w^a$  of the function approximator:

$$w_t^{a_t} = w_t^{a_t} + \eta_t \cdot N^k(d_t) \cdot \nabla Q(x_t; w_t^{a_t})$$

with the abbreviation

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{a_t})$$

- $\{x_i, x_{i+1}$  respectively denoting a state in the state space X,
- $\{a_i$  denoting an action from an action space A,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t^{a_t}$  denoting the weighting vector associated with the action  $a_t$  before the adaptation step,
- $w_{t+1}^{a_t}$  denoting the weighing vector associated with the action  $a_t$  after the adaptation step,
- $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in \{-1; 1\}$  denoting a risk monitoring parameter,
- $N^k$  denoting a risk monitoring function  $N^k(\xi) = (1 - \kappa \text{sign}(\xi)) \xi$  ( $= (1 - (\text{sign}(\xi))) \xi$ ),
- $\nabla Q(\cdot; \cdot)$  denoting the derivation of the function approximator according to its weights, and
- $r(x_i, a_i, x_{i+1})$  denoting a gain upon the transition of state from the state  $x_i$  to the subsequent state  $x_{i+1}$ .

The optimization function is preferably formed in accordance with the following [rule within the TD( $\lambda$ )-learning method:

$$\text{OFTD} = J(x; w)$$

- $\{x\}$  denoting a state in a state space  $X$ ,
- $\{a\}$  denoting an action from an action space  $A$ , and
- $\{w\}$  denoting the weights of a function approximator.

The following adaptation step is executed during TD( $\lambda$ -learning) in order to determine the optimum weights  $w$  of the function approximator:

$$w_{t+1} = w_t + \eta_{\{t\}} \gamma^* z_t$$

with the abbreviations

$$d_t = r(w_t, a_t, x_{t+1}) + \gamma J(x_{t+1}; w_t) - J(x_t; w_t),$$

$$z_t = \lambda \{ \gamma^* \}^\kappa d_t + z_{t-1} + \nabla J(x_t; w_t),$$

$$z_1 = 0$$

- $\{x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,
- $\{a_t\}$  denoting an action from an action space  $A$ ,
- $\gamma$  denoting a prescribable reduction factor,
- $\{w_t\}$  denoting the weighting vector before the adaptation step,
- $\{w_{t+1}\}$  denoting the weighting vector after the adaptation step,
- $\eta_{\{t\}}$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in \{-1; 1\}$  denoting a risk monitoring parameter,
- $N^*$  denoting a risk monitoring function  $N^*(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,  $\{() = (1 - \text{sign}(0))\}$ ,
- $\nabla J(*, *)$  denoting the derivation of the function approximator according to its [ ]weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

#### SUMMARY OF THE INVENTION

**It is an object of the present invention to provide a technical system and method for determining a] {The system is preferably a technical system of which before the determination measured values are measured which are used in determining the} sequence of actions [using measured values.**

**It is another object of the present invention to provide a technical system and method that]{.**

**The technical system} can be subjected to open-loop control or {else} closed-loop control with the use of {the} [a] determined sequence of actions.**

**{The system is preferably} [It is a further object of the invention to provide a technical system and method] modeled as a Markov {decision problem} [Decision Problem.]**

**{The method or the arrangement is preferably} [It is an additional object of the invention to provide a technical system and method that can be] used in a traffic management system {or} [.**

**It is yet another object of the invention to provide a technical system and method that can be used] in a communications system, {the} [such that a] sequence of actions {being used in a communications network} [is used] to carry out access control {or a routing, that is to say a path allocation,}, routing or path allocation.]**

**{Furthermore, the system can be a financial market which is modeled by a Markov decision problem, the change in the financial market, for example the} [It is yet a further object of the invention to provide a technical system and method for a financial market modeled by a Markov Decision Problem, wherein a] change in an {  
} index of stocks[,] or {else} [a change in] a rate of exchange on a foreign exchange market {being analyzed by using the method and/or the arrangement and it being}[, makes it ]possible to intervene in the market in accordance with {the} [a] sequence of determined actions.**

[These and other objects of the invention will be apparent from a careful review of the following detailed description of the preferred embodiments, which is to read in conjunction with a review of the accompanying drawing figures.

**BRIEF DESCRIPTION OF THE DRAWINGS** {Exemplary embodiments of the invention are illustrated in the figures and explained in more detail below.}

Figure 1 shows a flowchart {in which individual} [of] method steps {of the first exemplary embodiment are illustrated} [according to the present invention];

Figure 2 shows a {sketch-of-a} system {which can be} modeled as a Markov {decision problem} [Decision Problem];

Figure 3 shows a {sketch-of-a} communications network {in which} [wherein] access control is carried out in a switching unit [according to the present invention];

Figure 4 shows a {symbolic sketch-of-a} function approximator {with the aid of which a method of} [for] approximative dynamic programming {is implemented} [according to the present invention];

Figure 5 shows {a further sketch-of} a plurality of function approximators {with the aid of which} [for] approximative dynamic programming {is implemented} [according to the present invention]; and

Figure 6 shows a {sketch-of-a} traffic management system {which is} subjected to closed-loop control in accordance with {an exemplary embodiment.} [the present invention.]

**{First exemplary embodiment: access control and routing.} [DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS]**

{Figure 3 shows }[Figure 1 shows a flowchart according to the present invention, in which individual method steps of a first embodiment are provided, which will be discussed later.

Figure 2 shows the structure of a typical Markov Decision Problem method.

The system 201 is in a state  $x_t$  at an instant  $t$ . The state  $x_t$  can be observed by an observer of the system. On the basis of an action  $a_t$  from a set in the state  $x_t$  of possible actions,  $a_t \in A(x_t)$ , the system makes a transition with a certain probability into a subsequent state  $x_{t+1}$  at a subsequent instant  $t+1$ .

As illustrated diagrammatically in Figure 2 by a loop, an observer 200 perceives 202 observable variables concerning the state  $x_t$  and takes a decision via an action 203 with which it acts on the system 201. The system 201 is usually subject to the interference 205.

The observer 200 obtains a gain  $r_t$  204

$$r_t = r(x_t, a_t, x_{t+1}) \in \mathcal{R}, \quad (1)$$

which is a function of the action  $a_t$  203 and the original state  $x_t$  at the instant  $t$  as well as of the subsequent state  $x_{t+1}$  of the system at the subsequent instant  $t+1$ .

The gain  $r_t$  can assume a positive or negative scalar value depending on whether the decision leads, with regard to a prescribable criterion, to a positive or negative system development, to an increase in capital stock or to a loss.

In a further time step, the observer 200 of the system 201 decides on the basis of the observable variables 202, 204 of the subsequent state  $x_{t+1}$  in favor of a new action  $a_{t+1}$ , etc.

A sequence of

State:	$x_t$	$\in$	$X$
Action:	$a_t$	$\in$	$A(x_t)$
Subsequent state:	$x_{t+1}$	$\in$	$X$
Gain	$r_t = r(x_t, a_t, x_{t+1})$	$\in$	$\mathcal{R}$

describes a trajectory of the system which is evaluated by a performance criterion which accumulates the individual gains  $r_t$  over the instants  $t$ . It is assumed by way of simplification in a Markov Decision Problem that the state  $x_t$  and the action  $a_t$  all contain information for the purpose of describing a transition probability  $p(x_{t+1} | \cdot)$  of the system from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

In formal terms, this means that:

$$p(x_{t+1} | x_t, K, x_0, a_t, K, a_0) = p(x_{t+1} | x_t, a_t). \quad (2)$$

$p(x_{t+1} | x_t, a_t)$  denotes a transition probability for the subsequent state  $x_{t+1}$  for a given state  $x_t$  and given action  $a_t$ .

In a Markov Decision Problem, future states of the system 201 are thus not a function of states and actions which lie further in the past than one time step.

Figure 3 shows an embodiment of the present invention involving an access control and routing system, such as] a communications network 300[,-which]{.

The communications network 300] has a multiplicity of switching units 301a, 301b, ..., 301i, ... 301n, which are interconnected via connections 302a, 302b, 302j, ... 302m. [A] {Furthermore,-a} first terminal 303 is connected to a first switching unit 301a. From the first terminal 303, the first switching unit 301a is sent a request message 304 which requests preservation of a prescribed bandwidth within the communications network 300 for the purpose of transmitting data{, such as] video data{} [or] text data{}}.

It is determined in the first switching unit 301a in accordance with a strategy described below[,] whether the requested bandwidth is available in the {communications} [communi-cations] network 300 on a specified, requested connection {[step 305]} [in step 305]. {  
}The request is refused {[step 306]} [in step 306] if this is not the case. {

}If sufficient bandwidth is available, it is checked in {a further} checking step {{step 307}} [307] whether the bandwidth can be reserved.

The request is refused {{step 308}} [in step 308] if this is not the case. {  
}Otherwise, the first switching unit 301a selects a route from the first switching unit 301a via further switching units 301i to a second terminal 309 with which the first terminal 303 wishes to communicate, and a connection is initialized {{step 310}} [in step 310].

The starting point below is a communications network 300 which comprises a set of switching units

$$N = \{1, K_1, n, K_n, N\} \quad (17)$$

and a set of physical connections

$$L = \{1, K_1, 1, K_n, L\}, \quad (18)$$

a physical connection  $l$  having a capacity of  $B(l)$  bandwidth units.

A set

$$M = \{1, K_1, m, K_m, M\} \quad (19)$$

of different types of service  $m$  are available, a type of service  $m$  being characterized by

- {{a bandwidth requirement  $b(m)$ }},
- {{an average connection time  $\frac{1}{V(m)}$ , [ ] and}}
- {{a gain  $c(m)$  which is obtained whenever a call request of the corresponding type of service  $m$  is accepted.}}

The gain  $c(m)$  is given by the amount of money which a network operator of the communications network 300 bills a subscriber for a connection of the type of service. Clearly, the gain  $c(m)$  reflects different priorities, which can be prescribed by the network operator and which he associates with different services.

A physical connection 1 can simultaneously provide any desired combination of communications connections as long as the bandwidth used for the communications connections does not exceed the bandwidth available overall for the physical connection.

If a new communications connection of type m is requested between a first node i and a second node j (terminals are also denoted as nodes), the requested communications connection can, as represented above, either be accepted or be refused. {

}If the communications connection is accepted, a route is selected from a set of prescribed routes. This selection is denoted as a routing. b(m) bandwidth units are used in the communications connection of type m for each physical connection along [

]the selected route for the duration of the connection.

Thus, during access control{}, also referred to as] call admission control{}}, a route can be selected within the communications network 300 only when the selected route has sufficient bandwidth available. {

}The aim of the access control and of the routing is to maximize a long term gain which is obtained by acceptance of the requested connections.

At an instant t, the technical system which is the communications network 300 is in a state  $x_t$ , which is described by a list of routes via existing connections, by means of which lists it is shown how many connections of which type of service are using the respective routes at the instant t.

Events w, by means of which a state  $x_t$  could be transferred into a subsequent state  $x_{t+1}$ , are the arrival of new connection request messages, or else the {termination} [termina-tion] of a connection existing in the communications network 300.

In this {exemplary} embodiment, an action  $a_t$  at an instant  $t$ [,] owing to a connection request is the {

}decision as to whether a connection request is to be accepted or refused and, if the connection is accepted, the selection of the route through the communications network 300.

The aim is to determine a sequence of actions, that is to say clearly to determine the learning of a strategy with actions relating to a state  $x_i$  in such a way that the following rule is maximized:

$$E \left( \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}) \right), \quad (20)$$

- $\{E(\cdot)\}$  denoting an expectation,
- $t_k$  denoting an instant at which a  $k$ th event takes place,
- $g(x_{t_k}, \omega_k, a_{t_k})$  denoting the gain which is associated with the  $k$ th event, and
- $\beta$  denoting a reduction factor which evaluates an immediate gain as being more valuable than a gain at instants lying further in the future.

Different implementations of a strategy lead normally to different overall gains  $G$ :

$$G = \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}). \quad (21)$$

The aim is to maximize the expectation of the overall gain  $G$  in accordance with the following rule J:

$$J = E \left\{ \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}) \right\}, \quad (22)$$

it being possible to set a risk which reduces the overall gain  $G$  of a specific implementation of access control and of a routing strategy to below the expectation.

The TD( $\lambda$ )-learning method is used to carry out the access control and the [  
]routing.{  
}  
}

The following target function is used in this (exemplary) embodiment:

$$J^*(x_t) = E_{\tau} \left\{ e^{-\beta \tau} \right\} E_{\omega} \left\{ \max_{a \in A} [g(x_t, \omega_t, a) + J^*(x_{t+1})] \right\}, \quad (23)$$

- $\{\}$ A denoting an action space with a prescribed number of actions which are respectively available in a state  $x_t$ ,
- $\tau$   $\{\}$ denoting a first instant at which a first event  $\omega$   $\{\}$ occurs, and
- $\{\}$  $x_{t+1}$  denoting a subsequent state of the system.

An approximated value of the target value  $J^*(x_t)$  is learned and stored by employing a function approximator 400 (compare Figure 4) with the use of training data.

Training data are data previously measured in the communications network 300 and relating to the behavior of the communications network 300 in the case of incoming connection requests 304 and of termination of messages. This time sequence of states is stored, and these training data are used to train the function approximator 400 in accordance with the learning method described below.

A number of connections of in each case one type of service  $m$  on a route of the communications network 300 serve in each case as input variable of the function approximator 400 for each input 401, 402, 403 of the function approximator 400. [  
These are represented  $\{\}$ symbolically $\}$  in Figure 4 by blocks 404, 405, 406. {  
An approximated target value  $\tilde{J}$   $\{\}$  of the target value  $J$   $\}$  is the output variable of the function approximator 400.

Figure 5 shows a detailed representation of  $\{\}$ the $\}$  [a] function approximator 500, which  $\{\$ in this case $\}$  has several component function approximators 510, 520  $\{\$ of the function approximator 500 $\}$ . [

]One output variable is the approximated target value  $\tilde{J}$   $\{\}$ , which is formed in accordance with the following rule:

$$\tilde{J}(x_t, \Theta) = \sum_{l=1}^L \tilde{J}^{(l)}(x_t^{(l)}, \Theta_t^{(l)}) . \quad (24)$$

The input variables of the component function approximators 510, 520, which are present at the inputs 511, 512, 513 of the first component function approximator 510, or at the inputs 521, 522 and 523 of the second component function approximator 520 are, in turn, respectively a number of types of service of a type m in a physical connection r in each case, symbolized by blocks 514, 515, 516 for the first component function approximator, and 524, 525 and 526 for the second component function approximator 520.

Component output variables 530, 531, 532, 533 are fed to an adder unit 540, and the approximated target variable  $\tilde{J}$  is formed as output variable of the adder unit.

Let it be assumed that the communications network 300 is in the state  $x_{t_k}$  and that a request message with which a type of service m of class m is requested for a connection { } between two nodes i, j reaches the first switching unit 301a.

A list of permitted routes between the nodes i and j is denoted by  $R(i, j)$ , and a list of all possible routes is denoted by

$$\tilde{R}(i, j, x_{t_k}) \subset R(i, j) \quad (25)$$

as a subset of the routes  $R(i, j)$  which could implement a possible connection with regard to the available and requested bandwidth.

For each possible route  $r$ ,  $r \in \tilde{R}(i, j, x_{t_k})$ , a subsequent state  $x_{t_k+1}(x_{t_k}, \omega_k, r)$  is determined which results from the fact that the connection request 304 is accepted and the connection on the route  $r$  is made available to the requesting first terminal 303.

This is illustrated in Figure 1 as {a-second} step {{step-102}} [102], the state of the system and the respective event being respectively determined in {a-first} step {{step-101}} [101]. {

}A route  $r^*$  to be selected is determined in {a-third} step {{step-103}} [103] in accordance with the following rule:

$$r^* = \arg \max_{r \in \tilde{R}(i, j, x_{t_k})} \tilde{J}(x_{t_k+1}(x_{t_k}, \omega_k, r), \Theta_t). \quad (26)$$

A check is made in {a-further} step {{step-104}} [104] as to whether the following rule is fulfilled:

$$c(m) + \tilde{J}(x_{t_k+1}(x_{t_k}, \omega_k, r^*), \Theta_t) < \tilde{J}(x_{t_k}, \Theta_t). \quad (27)$$

If this is the case, the connection request 304 is rejected {{step-105}} [in step 105], otherwise the connection is accepted and \*switched through\* to the node  $j$  along the selected route  $r^*$  {{step-106}} [in step 106].

Weights of the function approximator 400, 500 which are adapted in the TD( $\lambda$ )]-learning method to the training data, are stored in a parameter vector  $\theta$  {{for an instant t in each case, such that an optimized access control and an optimized routing are achieved.}}

During the training phase, the weighting parameters are adapted to the training data applied to the function approximator.

A risk parameter  $\kappa$  {{is defined with the aid of which a desired risk, which the system has with regard to a prescribed state owing to a sequence of actions and states, can be set in accordance with the following rules:}}

- 1 ≤  $\kappa$  {{}} < 0: risky learning,
- $\kappa$  {{}} = 0: neutral learning with regard to the risk,
- 0 <  $\kappa$  {{}} < 1: risk-avoiding learning,
- $\kappa$  {{}} = 1: worst-case learning.

Furthermore, a prescribable parameter  $0 \leq \lambda \leq 1$  and a step size sequence  $\gamma_{tk}$  are prescribed in the learning method.

The weighting values of the weighting vector  $\Theta$  are adapted to the training data on the basis of each event  $\omega_{tk}$  in accordance with the following adaptation rule:

$$\Theta_k = \Theta_{k-1} + \gamma_k n^k(d_k) z_t, \quad (28)$$

in which case

$$d_k = e^{-\beta(t_k - t_{k-1})} (g(x_{t_k}, \omega_k, a_{t_k}) + \tilde{J}(x_{t_k}, \Theta_{k-1})) - \tilde{J}(x_{t_{k-1}}, \Theta_{k-1}) \quad (29)$$

$$z_t = \lambda e^{-\beta(t_{k-1} - t_{k-2})} z_{t-1} + \nabla_{\Theta} \tilde{J}(x_{t_{k-1}}, \Theta_{k-1}), \quad (30)$$

and

$$n^k(\xi) = (1 - \kappa \text{sign}(\xi)) \xi. \quad (31)$$

It is assumed that:  $z_1 = 0$ .

The function

$$g(x_{t_k}, \omega_k, a_{t_k}) \quad (32)$$

denotes the immediate gain in accordance with the following rule:

]

Thus, as described above, a sequence of actions is determined with regard to a connection request such that a connection request is either rejected or accepted on the basis of an action. The determination is performed taking account of an optimization function in which the risk can be set by means of a risk control parameter  $\kappa \in [-1; 1]$  in a variable fashion.

{Second exemplary embodiment: Traffic management system} [Figure 6 shows an embodiment of the present invention in relation to a traffic management system]

{Figure 6 shows a-[A] road 600 on which automobiles 601, 602, 603, 604, 605 and 606 are

being [

]driven.{

} Conductor loops 610, 611 integrated into the road 600 receive electric signals in a known way and feed the electric signals 615, 616 to a computer 620 via an input/output interface 621. In an analog-to-digital converter 622 connected to the input/output interface 621, the electric signals are digitized into a time series and stored in a memory 623, which is connected by a bus 624 to the analog-to-digital converter 622 and a processor 625. Via the input/output interface 621, a traffic management system 650 is fed control signals 651 from which it is possible to set a prescribed speed stipulation 652 in the traffic management system 650, or else further particulars of traffic regulations, which are displayed via the traffic management system 650 to drivers of the vehicles 601, 602, 603, 604, 605 and 606.

The following local state variables are used in this case for the purpose of traffic modeling:

- {()} traffic flow rate v,
- {()} vehicle density  $\rho$  ( $\rho$  = number of vehicles per kilometer  $\frac{Fz}{km}$  ).
- {()} traffic flow q ( $q$  = number of vehicles per hour  $\frac{Fz}{h}$  ), ( $q = v * \rho$  ), and
- {()} speed restrictions 652 displayed by the traffic management system 650 at an instant in each case.

The local state variables are measured as described above by using the conductor loops 610, 611.

These variables ( $v(t)$ ,  $\rho(t)$ ,  $q(t)$ ) therefore represent a state of the technical system of "traffic" at a specific instant t.

In this {exemplary} embodiment, the system is therefore a traffic system which is controlled by using the traffic management system 650[, and]{-

In this second exemplary embodiment, } an extended Q-learning method is described as method of approximative dynamic programming.

The state  $x_t$  is described by a state vector

$$x(t) = (v(t), p(t), q(t)). \quad (34)$$

The action  $a_t$  denotes the speed restriction 652, which is displayed at the instant t by the traffic management system 650. {

}The gain  $r(x_t, a_t, x_{t+1})$  describes the quality of the traffic flow which was measured between the instants t and t+1 by the conductor loops 610 and 611. [

]In this {second-exemplary} embodiment,  $r(x_t, a_t, x_{t+1})$  denotes

- {the average speed of the vehicles in the time interval [t, t + 1]}  
or
- {the number of vehicles which have passed the conductor loops 610 and 611 in the time interval [t, t + 1]}
  
- or
- {the variance of the vehicle speeds in the time interval [t, t + 1]},  
or
- {a weighted sum from the above variables.}

A value of the optimization function OFQ is determined for each possible action  $a_t$ , that is to say for each speed restriction which can be displayed by the traffic management system 650, an estimated value of the optimization function OFQ being realized in each case as a neural network.

This results in a set of evaluation variables for the various actions  $a_t$  in the [

]system state  $x_t$ {

} Those actions  $a_t$  for which the maximum evaluation variable OFQ has been determined in the current system state  $x_t$  are selected in a control phase from the possible actions  $a_t$ , that is to say from the set of the speed restrictions which can be displayed by the traffic management system 650.

In accordance with this {exemplary} embodiment, the adaptation rule, known from the Q-learning method, for calculating the optimization function OFQ is extended by a risk control function  $N^*(\theta(\cdot))$ , which takes account of the risk.

In turn, the risk control parameter  $\kappa(\cdot)$  is prescribed in accordance with the strategy from the first exemplary embodiment in the interval of  $[-1 \leq \kappa \leq 1]$ , and represents the risk which a user wishes to run in the application with regard to the control strategy to be determined.

The following evaluation function OFQ is used in accordance with this exemplary embodiment:

$$OFQ = Q(x; w^a), \quad (35)$$

- $\{x = (v; p(\cdot); q)$  denoting a state of the traffic system,
- $\{a$  denoting a speed restriction from the action space A of all speed restrictions which can be displayed by the traffic management system 650, and
- $\{w^a$  denoting the weights of the neural network which belong to the speed restriction a.

The following adaptation step is executed in Q-learning in order to determine [  
]the optimum weights  $w^a$  of the neural network:

$$w_{t+1}^{at} = w_t^{at} + \eta_t \cdot N^*(d_t) \cdot \nabla Q(x_t; w_t^{at}) \quad (36)$$

using the abbreviation[:]

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{at}) \quad (37)$$

- $\{x_t, x_{t+1}$  denoting in each case a state of the traffic system in accordance with rule (34),

- $a_i$  denoting an action, that is to say a speed restriction which can be displayed by the traffic management system 650,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t^{a_i}$  denoting the weighting vector belonging to the action  $a_i$ , before the adaptation step,
- $w_{t+1}^{a_i}$  denoting the weighting vector belonging to the action  $a_i$ , after the adaptation step,
- $\eta_{t+1}$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk control parameter,
- $N^*$  denoting a risk control function  $N^*(\xi) = (1 + \kappa \text{sign}(\xi))\xi$ ,
- $\nabla_a(\cdot)$  denoting the derivative of the neural network with respect to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition in state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

An action  $a_i$  can be selected at random from the possible actions  $a_i$  during learning. It is not necessary in this case to select the action  $a_i$  which has led to the largest evaluation variable.

The adaptation of the weights has to be performed in such a way that not only is a traffic control achieved which is optimized in terms of the expectation of the optimization function, but that also account is taken of a variance of the control results.

This is particularly advantageous since the state vector  $x(t)$  models the actual system of traffic only inadequately in some aspects, and so unexpected disturbances can thereby occur. Thus, the dynamics of the traffic, and therefore of its modeling, depend on further factors such as weather, proportion of trucks on the road, proportion of mobile homes, etc., which are not always integrated in the measured variables of the state vector  $x(t)$ . In addition, it is not always ensured that the road users immediately implement the new speed instructions in accordance with the traffic management system.

A control phase on the real system in accordance with the traffic management system takes place in accordance with the following steps:

1. The state  $x_i$  is measured at the instant  $t$  at various points in the traffic system of traffic and yields a state vector  $x(t) = (v(t), p(\cdot)(t), q(t))$ .
2. A value of the optimization function is determined for all possible actions  $a_i$ , and that action  $a_i$  with the highest evaluation in the optimization function is selected.

#### Abstract

Method and arrangement for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action

The determination of a sequence of actions is performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function. The optimization function includes a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system; [Although modifications and changes may be suggested by those skilled in the

art to which this invention pertains, it is the intention of the inventors to embody within the patent warranted hereon all changes and modifications that may reasonably and properly come under the scope of their contribution to the art. - -]

**Description**

Method and arrangement for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action

The invention relates to a method and an arrangement for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action.

Such a method and such an arrangement are known from [1].

A financial market is described in [1] as an example for such a system which has states.

The system is described as a Markov decision problem (MDP). The structure of a system which can be described as a Markov decision problem is illustrated in Figure 2.

The system 201 is in a state  $x_t$  at an instant  $t$ . The state  $x_t$  can be observed by an observer of the system. On the basis of an action  $a_t$  from a set in the state  $x_t$  of possible actions,  $a_t \in A(x_t)$ , the system makes a transition with a certain probability into a subsequent state  $x_{t+1}$  at a subsequent instant  $t+1$ .

This is illustrated diagrammatically in Figure 2 by a loop. An observer 200 perceives 202 observable variables concerning the state  $x_t$  and takes a decision via an action 203 with which it acts on the system 201.

The system 201 is usually subject to the interference 205.

Furthermore, the observer 200 obtains a gain  $r_t$

204

$$r_t = r(x_t, a_t, x_{t+1}) \in \mathbb{R}, \quad (1)$$

which is a function of the action  $a_t$  at 203 and the original state  $x_t$  at the instant  $t$  as well as of the 5 subsequent state  $x_{t+1}$  of the system at the subsequent instant  $t+1$ .

The gain  $r_t$  can assume a positive or negative skalar value, depending on whether the decision leads, with regard to a prescribable criterion, to a positive 10 or negative system development, in [1] to an increase in capital stock or to a loss.

In a further time step, the observer 200 of the system 201 decides on the basis of the observable variables 202, 204 of the subsequent state  $x_{t+1}$  in favor 15 of a new action  $a_{t+1}$ , etc.

A sequence of

State:	$x_t$	$\in$	$X$
Action:	$a_t$	$\in$	$A(x_t)$
Subsequent state:	$x_{t+1}$	$\in$	$X$
Gain	$r_t = r(x_t, a_t, x_{t+1})$	$\in$	$\mathbb{R}$

etc. describes a trajectory of the system which is evaluated by a performance criterion which accumulates 25 the individual gains  $r_t$  over the instants  $t$ . It is assumed by way of simplification in a Markov decision problem that the state  $x_t$  and the action  $a_t$  all contain information for the purpose of describing a transition probability  $p(x_{t+1}|\cdot)$  of the system from the state  $x_t$  to 30 the subsequent state  $x_{t+1}$ .

In formal terms, this means that:

$$p(x_{t+1}|x_t, K, x_0, a_t, K, a_0) = p(x_{t+1}|x_t, a_t). \quad (2)$$

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$p(x_{t+1}|x_t, a_t)$  denotes a transition probability for the subsequent state  $x_{t+1}$  for a given state  $x_t$  and given action  $a_t$ .

In a Markov decision problem, future states of the system 201 are thus not a function of states and actions which lie further in the past than one time step.

The characteristics of a Markov decision problem are represented below by way of summary:

10

$X$  set of possible states of the system,  
e.g.  $X = \mathbb{R}^m$ ,

$A(x_t)$  set of possible actions in the state  
 $p(x_{t+1}|x_t, a_t)$   $x_t$

15

$r(x_t, a_t, x_{t+1})$  gain with expectation  $R(x_t, a_t)$ .

Starting from observable variables, the variables denoted below as training data, the aim is to determine a strategy, that is to say a sequence of 20 functions

$$\pi = \{\mu_0, \mu_1, K, \mu_T\}, \quad (3)$$

which at each instant  $t$  map each state into an action rule, that is to say action

$$\mu_t(x_t) = a_t \quad (4)$$

25

Such a strategy is evaluated by an optimization function.

The optimization function specifies the expectation, the gains accumulated over time at a given strategy  $\pi$ , and a start state  $x_0$ .

5 The so-called Q-learning method is described in [1] as an example of a method of approximative dynamic programming.

An optimum evaluation function  $V^*(x)$  is defined by

$$V^*(x) = \max_{\pi} V^{\pi}(x) \quad \forall x \in X \quad (5)$$

10 where

$$V^{\pi}(x) = E \left[ \sum_{t=0}^{\infty} \gamma^t r(x_t, \mu_t, x_{t+1}) | x_0 = x \right], \quad (6)$$

$\gamma$  denoting a prescribable reduction factor which is formed in accordance with the following rule:

$$\gamma = \frac{1}{1+z}, \quad (7)$$

$$z \in \mathbb{R}^+. \quad (8)$$

15 A Q-evaluation function  $Q^*(x_t, a_t)$  is formed within the Q-learning method for each pair (state  $x_t$ , action  $a_t$ ) in accordance with the following rule:

$$Q^*(x_t, a_t) = \sum_{x \in X} p(x_{t+1}|x_t, a_t) \cdot r_t + \\ + \gamma \cdot \sum_{x \in X} p(x|x_t, a_t) \cdot \max_{a \in A} (Q^*(x, a))$$

(9)

- 5 -

On the basis respectively of the tuple  $(x_t, x_{t+1}, a_t, r_t)$ , the Q-values  $Q^*(x, a)$  are adapted in the  $k+1$  th iteration in accordance with the following learning rule with a prescribed learning rate  $\eta_k$  in accordance with the following rule:

$$Q_{k+1}(x_t, a_t) = (1 - \eta_k)Q_k(x_t, a_t) + \eta_k \left( r_t + \gamma \max_{a \in A} (Q_k(x_{t+1}, a)) \right). \quad (10)$$

Usually, the so-called Q-values  $Q^*(x, a)$  are approximated for various actions  $a$  by a function approximator in each case, for example a neural network or else a polynomial classifier, with a weighting vector  $w^a$ , which contains weights of the function approximator.

A function approximator is to be understood as, for example, a neural network, a polynomial classifier or else a combination of a neural network with a polynomial classifier.

It therefore holds that:

$$Q^*(x, a) \approx q(x; w^a). \quad (11)$$

Changes in the weights in the weighting vector  $w^a$  are based on a temporal difference  $d_t$  which is formed in accordance with the following rule:

$$d_t := r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} q(x_{t+1}; w_k^a) - q(x_t; w_k^{a_t}) \quad (12)$$

The following adaptation rule for the weights of the neural network, which are included in the weighting vector  $w^a$ , follows for the Q-learning method with the use of a neural network:

$$w_{k+1}^{at} = w_k^{at} + \eta_k \cdot d_t \cdot \nabla Q(x_t; w_k^{at}). \quad (13)$$

The neural network representing the system of a financial market, as described in [1], is trained using the training data which describe information on changes 5 in prices on a financial market as time series values.

A further method of approximative dynamic programming, the so-called TD( $\lambda$ )-learning method, is known from [2] and is explained in more detail in conjunction with an exemplary embodiment.

10 Furthermore, it is known from [3] which risk is associated with a strategy  $\pi$  and an initial state  $x_t$ . A method for risk avoidment is likewise known from [3].

15 The following optimization function, which is also referred to as an expanded Q-function  $\underline{Q}^*(x_t, a_t)$ , is used in the method known from [3]:

$$\begin{aligned} & \text{maximize} \\ & \left\{ \underline{Q}^*(x_t, a_t) = r(x_t, a_t, x_{t+1}) + \inf_{\substack{x_0, x_1, K \\ p(x_0, x_1, K) > 0}} \left\{ \sum_{k=1}^{\infty} \gamma^k r(x_k, \pi(x_k), x_{k+1}) \right\} \right\} \end{aligned} \quad (14)$$

The expanded Q-function  $\underline{Q}^*(x_t, a_t)$  describes the worst case if the action  $a_t$  is executed in the state  $x_t$  20 and the strategy  $\pi$  is followed thereupon.

The optimization function  $\underline{Q}^*(x_t, a_t)$  for

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$$\underline{Q}^*(x_t, a_t) := \max_{\pi \in \Pi} \underline{Q}^\pi(x_t, a_t)$$

(15)

is given by the following rule:

$$\underline{Q}^*(x_t, a_t) = \min_{x \in X} \left[ r(x_t, a_t, x) + \gamma \cdot \max_{a \in A} \underline{Q}^*(x, a) \right]. \quad (16)$$

$p(x_{t+1}|x_t, a_t) > 0$

5        A substantial disadvantage of this mode of procedure is to be seen in that only the worst case is taken into account when finding the strategy. However, this reflects the requirements of the most varied technical systems only to an inadequate extent.

10      Furthermore, it is known from [4] to formulate access control for a communications network and the routing within the communications network as a problem of dynamic programming.

15      The invention is therefore based on the problem of specifying a method and an arrangement for determining a sequence of actions for a system, in which method or action an increased flexibility in determining the strategy is achieved.

20      The problem is solved by the method and by the arrangement in accordance with the features of the independent patent claims.

25      In a method for computer-aided determination of a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, the determination of the sequence of actions is performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization

function including a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

5        An arrangement for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action, has a processor which is set up in such a way that the determination of the sequence of actions  
10      can be performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function, the optimization function including a variable parameter with the aid of which it is possible  
15      to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

It becomes possible for the first time owing to the invention to specify a method for determining a sequence of actions at a freely prescribable level of  
20      accuracy when finding a strategy for a possible closed-loop control or open-loop control of the system, in general for influencing it.

Preferred developments of the invention follow from the dependent claims.  
25      The developments described below are valid both for the method and for the arrangement, the processor being respectively set up in the development of arrangement in such a way that the development can be implemented.

30      In a preferred refinement, a method of approximative dynamic programming is used for the purpose of determination, for example a method based on Q-learning or else a method based on TD( $\lambda$ )-learning.

Within Q-learning, the optimization function OFQ is preferably formed in accordance with the following rule:

$$\text{OFQ} = Q(x; w^a),$$

5   •    $x$  denoting a state in a state space  $X$   
   •    $a$  denoting an action from an action space  $A$ , and  
   •    $w^a$  denoting the weights of a function approximator which belong to the action  $a$ .

10   The following adaptation step is executed during Q-learning in order to determine the optimum weights  $w^a$  of the function approximator:

$$w_{t+1}^{at} = w_t^{at} + \eta_t \cdot N^k(d_t) \cdot \nabla Q(x_t; w_t^{at})$$

with the abbreviation

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{at})$$

15   •    $x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,  
   •    $a_t$  denoting an action from an action space  $A$ ,  
   •    $\gamma$  denoting a prescribable reduction factor,  
   •    $w_t^{at}$  denoting the weighting vector associated with  
     the action  $a_t$  before the adaptation step,  
   •    $w_{t+1}^{at}$  denoting the weighing vector associated with  
     the action  $a_t$  after the adaptation step,  
   •    $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,

- $\kappa \in [-1; 1]$  denoting a risk monitoring parameter,
- $N^*$  denoting a risk monitoring function  $N^*(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- 5 •  $\nabla Q(\cdot; \cdot)$  denoting the derivation of the function approximator according to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

10 The optimization function is preferably formed in accordance with the following rule within the  $TD(\lambda)$ -learning method:

OFTD =  $J(x; w)$

15

- $x$  denoting a state in a state space  $X$ ,
- $a$  denoting an action from an action space  $A$ , and
- $w$  denoting the weights of a function approximator.

20 The following adaptation step is executed during  $TD(\lambda)$ -learning in order to determine the optimum weights  $w$  of the function approximator:

$$w_{t+1} = w_t + \eta_t \cdot N^*(d_t) \cdot z_t$$

25 with the abbreviations

$$d_t = r(w_t, a_t, x_{t+1}) + \gamma J(x_{t+1}; w_t) - J(x_t; w_t),$$

$$30 z_t = \lambda \cdot \gamma \cdot z_{t-1} + \nabla J(x_t; w_t),$$

$$z_{-1} = 0$$

- $x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,
- $a_t$  denoting an action from an action space  $A$ ,
- 5 •  $\gamma$  denoting a prescribable reduction factor,
- $w_t$  denoting the weighting vector before the adaptation step,
- $w_{t+1}$  denoting the weighting vector after the adaptation step,
- 10 •  $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk monitoring parameter,
- $N^*$  denoting a risk monitoring function  $N^*(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- 15 •  $\nabla J(\cdot, \cdot)$  denoting the derivation of the function approximator according to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

20

The system is preferably a technical system of which before the determination measured values are measured which are used in determining the sequence of actions.

25

The technical system can be subjected to open-loop control or else closed-loop control with the use of the determined sequence of actions.

30

The system is preferably modeled as a Markov decision problem.

The method or the arrangement is preferably used in a traffic management system or in a communications system, the sequence of actions being used in a communications network to carry out access control or a routing, that is to say a path allocation.

35

Furthermore, the system can be a financial market which is modeled by a Markov decision problem, the change in the financial market, for example the change in an

index of stocks or else a rate of exchange on a foreign exchange market being analyzed by using the method and/or the arrangement and it being possible to intervene in the market in accordance with the sequence 5 of determined actions.

Exemplary embodiments of the invention are illustrated in the figures and explained in more detail below.

10 Figure 1 shows a flowchart in which individual method steps of the first exemplary embodiment are illustrated;

Figure 2 shows a sketch of a system which can be modeled as a Markov decision problem;

15 Figure 3 shows a sketch of a communications network in which access control is carried out in a switching unit;

20 Figure 4 shows a symbolic sketch of a function approximator with the aid of which a method of approximative dynamic programming is implemented;

25 Figure 5 shows a further sketch of a plurality of function approximators with the aid of which approximative dynamic programming is implemented; and

Figure 6 shows a sketch of a traffic management system which is subjected to closed-loop control in accordance with an exemplary embodiment.

**First exemplary embodiment: access control and routing.**

5       **Figure 3** shows a communications network 300, which has  
a multiplicity of switching units 301a, 301b, ...,  
301i, ... 301n, which are interconnected via  
connections 302a, 302b, 302j, ... 302m.

10      Furthermore, a first terminal 303 is connected  
to a first switching unit 301a. From the first terminal  
303, the first switching unit 301a is sent a request  
15     message 304 which requests preservation of a prescribed  
bandwidth within the communications network 300 for the  
purpose of transmitting data (video data, text data).

15      It is determined in the first switching unit  
301a in accordance with a strategy described below  
whether the requested bandwidth is available in the  
communications network 300 on a specified, requested  
connection (step 305).

20      The request is refused (step 306) if this is  
not the case.

20      If sufficient bandwidth is available, it is  
checked in a further checking step (step 307) whether  
the bandwidth can be reserved.

25      The request is refused (step 308) if this is  
not the case.

25      Otherwise, the first switching unit 301a  
selects a route from the first switching unit 301a via  
further switching units 301i to a second terminal 309  
with which the first terminal 303 wishes to  
communicate, and a connection is initialized (step  
30 310).

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The starting point below is a communications network 300 which comprises a set of switching units

$$N = \{1, K, n, K, N\} \quad (17)$$

and a set of physical connections

$$L = \{1, K, l, K, L\}, \quad (18)$$

5 a physical connection  $l$  having a capacity of  $B(l)$  bandwidth units.

A set

$$M = \{1, K, m, K, M\} \quad (19)$$

10 of different types of service  $m$  are available, a type of service  $m$  being characterized by

- a bandwidth requirement  $b(m)$ ,
- an average connection time  $\frac{1}{v(m)}$  and
- a gain  $c(m)$  which is obtained whenever a call request of the corresponding type of service  $m$  is accepted.

15 The gain  $c(m)$  is given by the amount of money which a network operator of the communications network 300 bills a subscriber for a connection of the type of service. Clearly, the gain  $c(m)$  reflects different priorities, which can be prescribed by the network operator and which he associates with different services.

20 25 A physical connection  $l$  can simultaneously provide any desired combination of communications connections as long as the bandwidth used for the communications connections does not exceed the bandwidth available overall for the physical connection.

30

If a new communications connection of type m is requested between a first node i and a second node j (terminals are also denoted as nodes), the requested communications connection can, as represented above,  
5 either be accepted or be refused.

If the communications connection is accepted, a route is selected from a set of prescribed routes. This selection is denoted as a routing.  $b(m)$  bandwidth units are used in the communications connection of type m for  
10 each physical connection along the selected route for the duration of the connection.

Thus, during access control (call admission control), a route can be selected within the communications network 300 only when the selected route  
15 has sufficient bandwidth available.

The aim of the access control and of the routing is to maximize a long term gain which is obtained by acceptance of the requested connections.

At an instant t, the technical system which is  
20 the communications network 300 is in a state  $x_t$  which is described by a list of routes via existing connections, by means of which lists it is shown how many connections of which type of service are using the respective routes at the instant t.

25 Events w, by means of which a state  $x_t$  could be transferred into a subsequent state  $x_{t+1}$ , are the arrival of new connection request messages, or else the termination of a connection existing in the communications network 300.

30 In this exemplary embodiment, an action at at an instant t owing to a connection request is the

decision as to whether a connection request is to be accepted or refused and, if the connection is accepted, the selection of the route through the communications network 300.

5        The aim is to determine a sequence of actions, that is to say clearly to determine the learning of a strategy with actions relating to a state  $x_t$  in such a way that the following rule is maximized:

$$\mathbb{E} \left( \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}) \right), \quad (20)$$

10      •      $E\{\cdot\}$  denoting an expectation,  
         •      $t_k$  denoting an instant at which a  $k$ th event takes place,  
         •      $g(x_{t_k}, \omega_k, a_{t_k})$ . denoting the gain which is associated with the  $k$ th event, and  
 15      •      $\beta$  denoting a reduction factor which evaluates an immediate gain as being more valuable than a gain at instants lying further in the future.

Different implementations of a strategy lead normally to different overall gains  $G$ :

$$G = \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}). \quad (21)$$

20      The aim is to maximize the expectation of the overall gain  $G$  in accordance with the following rule  $J$ :

$$J = E \left\{ \sum_{k=0}^{\infty} e^{-\beta t_k} \cdot g(x_{t_k}, \omega_k, a_{t_k}) \right\}, \quad (22)$$

it being possible to set a risk which reduces the overall gain G of a specific implementation of access control and of a routing strategy to below the  
5 expectation.

The TD( $\lambda$ )-learning method is used to carry out the access control and the routing.

The following target function is used in this exemplary embodiment:

$$J^*(x_t) = E_t \left\{ e^{-\beta t} \right\} E_{\omega} \left\{ \max_{a \in A} [g(x_t, \omega_t, a) + J^*(x_{t+1})] \right\}, \quad (23)$$

- 10 • A denoting an action space with a prescribed number of actions which are respectively available in a state  $x_t$ ,
- $t$  denoting a first instant at which a first event  
15  $\omega$  occurs, and
- $x_{t+1}$  denoting a subsequent state of the system.

An approximated value of the target value  $J^*(x_t)$  is learned and stored by employing a function approximator 400 (compare Figure 4) with the use of  
20 training data.

Training data are data previously measured in the communications network 300 and relating to the behavior of the communications network 300 in the case of incoming connection requests 304 and of termination  
25 of messages. This time sequence of states is stored, and these training data are used to train the function approximator 400 in accordance with the learning method described below.

ROTATION: 300 304 302 303 301 305

- 18 -

A number of connections of in each case one type of service  $m$  on a route of the communications network 300 serve in each case as input variable of the function approximator 400 for each input 401, 402, 403 5 of the function approximator 400. These are represented symbolically in Figure 4 by blocks 404, 405, 406.

An approximated target value  $\tilde{J}$  of the target value  $J^*$  is the output variable of the function approximator 400.

10 Figure 5 shows a detailed representation of the function approximator 500, which in this case has several component function approximators 510, 520 of the function approximator 500. One output variable is the approximated target value  $\tilde{J}$ , which is formed in 15 accordance with the following rule:

$$\tilde{J}(x_t, \Theta) = \sum_{l=1}^L \tilde{J}^{(l)}(x_t^{(l)}, \Theta_t^{(l)}). \quad (24)$$

The input variables of the component function approximators 510, 520, which are present at the inputs 511, 512, 513 of the first component function approximator 510, or at the inputs 521, 522 and 523 of the second component function approximator 520 are, in turn, respectively a number of types of service of a type  $m$  in a physical connection  $r$  in each case, symbolized by blocks 514, 515, 516 for the first 25 component function approximator, and 524, 525 and 526 for the second component function approximator 520.

Component output variables 530, 531, 532, 533 are fed to an adder unit 540, and the approximated target variable  $\tilde{J}$  is formed as output variable of the 30 adder unit.

Let it be assumed that the communications network 300 is in the state  $x_{tk}$  and that a request message with which a type of service  $m$  of class  $m$  is requested for a connection

- 19 -

between two nodes  $i, j$  reaches the first switching unit 301a.

A list of permitted routes between the nodes  $i$  and  $j$  is denoted by  $R(i, j)$ , and a list of all possible 5 routes is denoted by

$$\tilde{R}(i, j, x_{t_k}) \subset R(i, j) \quad (25)$$

as a subset of the routes  $R(i, j)$  which could implement a possible connection with regard to the available and requested bandwidth.

10 For each possible route  $r$ ,  $r \in \tilde{R}(i, j, x_{t_k})$ , a subsequent state  $x_{t_k+1}(x_{t_k}, \omega_k, r)$  is determined which results from the fact that the connection request 304 is accepted and the connection on the route  $r$  is made available to the requesting first terminal 303.

15 This is illustrated in Figure 1 as second step (step 102), the state of the system and the respective event being respectively determined in a first step (step 101).

20 A route  $r^*$  to be selected is determined in a third step (step 103) in accordance with the following rule:

$$r^* = \arg \max_{r \in \tilde{R}(i, j, x_{t_k})} \tilde{J}(x_{t_k+1}(x_{t_k}, \omega_k, r), \Theta_t). \quad (26)$$

A check is made in a further step (step 104) as to whether the following rule is fulfilled:

$$c(m) + \tilde{J}(x_{t_k+1}(x_{t_k}, \omega_k, r^*), \Theta_t) < \tilde{J}(x_{t_k}, \Theta_t). \quad (27)$$

25

- 20 -

If this is the case, the connection request 304 is rejected (step 105), otherwise the connection is accepted and "switched through" to the node j along the selected route  $r^*$  (step 106).

5       Weights of the function approximator 400, 500 which are adapted in the TD( $\lambda$ )-learning method to the training data, are stored in a parameter vector  $\Theta$  for an instant t in each case, such that an optimized access control and an optimized routing are achieved.

10      During the training phase, the weighting parameters are adapted to the training data applied to the function approximator.

15      A risk parameter  $\kappa$  is defined with the aid of which a desired risk, which the system has with regard to a prescribed state owing to a sequence of actions and states, can be set in accordance with the following rules:

-1 ≤  $\kappa$  < 0:     risky learning,  
 20     $\kappa = 0$ :     neutral learning with regard to the risk,  
       0 <  $\kappa$  < 1:   risk-avoiding learning,  
        $\kappa = 1$ :     worst-case learning.

25      Furthermore, a prescribable parameter  $0 \leq \lambda \leq 1$  and a step size sequence  $\gamma_k$  are prescribed in the learning method.

The weighting values of the weighting vector  $\Theta$  are adapted to the training data on the basis of each 30 event  $\omega_{tk}$  in accordance with the following adaptation rule:

$$\Theta_k = \Theta_{k-1} + \gamma_k N^{\kappa}(d_k) z_t, \quad (28)$$

in which case

$$d_k = e^{-\beta(t_k - t_{k-1})} (g(x_{t_k}, \omega_k, a_{t_k}) + \tilde{J}(x_{t_k}, \theta_{k-1})) - \tilde{J}(x_{t_{k-1}}, \theta_{k-1}) \quad (29)$$

$$z_t = \lambda e^{-\beta(t_{k-1} - t_{k-2})} z_{t-1} + \nabla_{\theta} \tilde{J}(x_{t_{k-1}}, \theta_{k-1}), \quad (30)$$

and

$$\kappa^*(\xi) = (1 - \kappa \text{sign}(\xi)) \xi. \quad (31)$$

5 It is assumed that:  $z_{-1} = 0$ .

The function

$$g(x_{t_k}, \omega_k, a_{t_k}) \quad (32)$$

denotes the immediate gain in accordance with the following rule:

$$10 g(x_{t_k}, \omega_k, a_{t_k}) = \begin{cases} c(m) & \text{when } \omega_{t_k} \text{ is a service request} \\ & \text{for a type of service } m, \text{ and the} \\ & \text{connection is accepted} \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

Thus, as described above, a sequence of actions is determined with regard to a connection request such that a connection request is either rejected or accepted on the basis of an action. The determination is performed taking account of an optimization function in which the risk can be set by means of a risk control parameter  $\kappa \in [-1; 1]$  in a variable fashion.

**Second exemplary embodiment: Traffic management system**

Figure 6 shows a road 600 on which automobiles 601, 602, 603, 604, 605 and 606 are being driven.

Conductor loops 610, 611 integrated into the road 600 receive electric signals in a known way and feed the electric signals 615, 616 to a computer 620 via an input/output interface 621. In an analog-to-digital converter 622 connected to the input/output interface 621, the electric signals are digitized into a time series and stored in a memory 623, which is connected by a bus 624 to the analog-to-digital converter 622 and a processor 625. Via the input/output interface 621, a traffic management system 650 is fed control signals 651 from which it is possible to set a prescribed speed stipulation 652 in the traffic management system 650, or else further particulars of traffic regulations, which are displayed via the traffic management system 650 to drivers of the vehicles 601, 602, 603, 604, 605 and 606.

The following local state variables are used in this case for the purpose of traffic modeling:

- traffic flow rate  $v$ ,
- vehicle density  $\rho$  ( $\rho$  = number of vehicles per kilometer  $\frac{Fz}{km}$ ).
- traffic flow  $q$  ( $q$  = number of vehicles per hour  $\frac{Fz}{h}$ , ( $q=v * \rho$ )), and
- speed restrictions 652 displayed by the traffic management system 650 at an instant in each case.

The local state variables are measured as described above by using the conductor loops 610, 611.

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These variables ( $v(t)$ ,  $p(t)$ ,  $q(t)$ ) therefore represent a state of the technical system of "traffic" at a specific instant  $t$ .

In this exemplary embodiment, the system is 5 therefore a traffic system which is controlled by using the traffic management system 650.

In this second exemplary embodiment, an extended Q-learning method is described as method of approximative dynamic programming.

10 The state  $x_t$  is described by a state vector

$$x(t) = (v(t), p(t), q(t)). \quad (34)$$

The action  $a_t$  denotes the speed restriction 652, which is displayed at the instant  $t$  by the traffic management system 650.

15 The gain  $r(x_t, a_t, x_{t+1})$  describes the quality of the traffic flow which was measured between the instants  $t$  and  $t+1$  by the conductor loops 610 and 611. In this second exemplary embodiment,  $r(x_t, a_t, x_{t+1})$  denotes

20 • the average speed of the vehicles in the time interval  $[t, t + 1]$

or

• the number of vehicles which have passed the conductor loops 610 and 611 in the time interval 25  $[t, t + 1]$

or

• the variance of the vehicle speeds in the time interval  $[t, t + 1]$ ,

or

- a weighted sum from the above variables.

5 A value of the optimization function OFQ is determined for each possible action  $a_t$ , that is to say for each speed restriction which can be displayed by the traffic management system 650, an estimated value of the optimization function OFQ being realized in each case as a neural network.

10 This results in a set of evaluation variables for the various actions  $a_t$  in the system state  $x_t$ .

Those actions  $a_t$  for which the maximum evaluation variable OFQ has been determined in the current system state  $x_t$  are selected in a control phase from the possible actions  $a_t$ , that is to say from the 15 set of the speed restrictions which can be displayed by the traffic management system 650.

20 In accordance with this exemplary embodiment, the adaptation rule, known from the Q-learning method, for calculating the optimization function OFQ is extended by a risk control function  $N^*(.)$ , which takes account of the risk.

In turn, the risk control parameter  $\kappa$  is prescribed in accordance with the strategy from the first exemplary embodiment in the interval of 25  $[-1 \leq \kappa \leq 1]$ , and represents the risk which a user wishes to run in the application with regard to the control strategy to be determined.

The following evaluation function OFQ is used in accordance with this exemplary embodiment:

$$30 \quad OFQ = Q(x; w^a), \quad (35)$$

5     •     $x = (v; p; q)$  denoting a state of the traffic system,

•    a denoting a speed restriction from the action space A of all speed restrictions which can be displayed by the traffic management system 650, and

•     $w^a$  denoting the weights of the neural network which belong to the speed restriction a.

10   The following adaptation step is executed in Q-learning in order to determine the optimum weights  $w^a$  of the neural network:

$$w_{t+1}^{at} = w_t^{at} + \eta_t \cdot N^K(d_t) \cdot \nabla Q(x_t; w_t^{at}) \quad (36)$$

using the abbreviation

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{at}) \quad (37)$$

15   •     $x_t$ ,  $x_{t+1}$  denoting in each case a state of the traffic system in accordance with rule (34),

•     $a_t$  denoting an action, that is to say a speed restriction which can be displayed by the traffic management system 650,

20   •     $\gamma$  denoting a prescribable reduction factor,

•     $w_t^{at}$  denoting the weighting vector belonging to the action  $a_t$ , before the adaptation step,

•     $w_{t+1}^{at}$  denoting the weighting vector belonging to the action  $a_t$ , after the adaptation step,

25   •     $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,

- $\kappa \in [-1; 1]$  denoting a risk control parameter,
- $N^*$  denoting a risk control function  $N^*(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- $\nabla_q(\cdot, \cdot)$  denoting the derivative of the neural network with respect to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition in state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

10 An action  $a_t$  can be selected at random from the possible actions  $a_t$  during learning. It is not necessary in this case to select the action  $a_t$  which has led to the largest evaluation variable.

15 The adaptation of the weights has to be performed in such a way that not only is a traffic control achieved which is optimized in terms of the expectation of the optimization function, but that also account is taken of a variance of the control results.

20 This is particularly advantageous since the state vector  $x(t)$  models the actual system of traffic only inadequately in some aspects, and so unexpected disturbances can thereby occur. Thus, the dynamics of the traffic, and therefore of its modeling, depend on further factors such as weather, proportion of trucks on the road, proportion of mobile homes, etc., which 25 are not always integrated in the measured variables of the state vector  $x(t)$ . In addition, it is not always ensured that the road users immediately implement the new speed instructions in accordance with the traffic management system.

30 A control phase on the real system in accordance with the traffic management system takes place in accordance with the following steps:

35 1. The state  $x_t$  is measured at the instant  $t$  at various points in the traffic system of traffic and yields a state vector  $x(t) = (v(t), p(t), q(t))$ .

- 27 -

2. A value of the optimization function is determined for all possible actions  $a_t$ , and that action  $a_t$  with the highest evaluation in the optimization function is selected.

The following publications are cited in this document:

5 [1] R. Neuneier, Enhancing Q-Learning for Optimal  
Asset Allocation, Proceedings of the Neural  
Information Processing Systems, NIPS 1997

10 [2] R.S. Sutton, Learning to predict by the method of  
temporal differences, Machine Learning, 3:9-44,  
1988

15 [3] M. Heger, Risk and Reinforcement Learning:  
Concepts and Dynamic Programming, ZKW Bericht Nr.  
8/94, Zentrum für Kognitionswissenschaften [Center  
for Cognitive Sciences], Bremen University, ISSN  
0947-0204, December 1994

[4] D.P. Bertsekas, Dynamic Programming and Optimal  
Control, Athena Scientific, Belmont, MA, 1995

**Patent claims**

1. A method for computer-aided determination of a  
5 sequence of actions for a system which has states, a  
transition in state between two states being performed  
on the basis of an action, in the case of which the  
determination of the sequence of actions is performed  
10 in such a way that a sequence of states resulting from  
the sequence of actions is optimized with regard to a  
prescribed optimization function, the optimization  
function including a variable parameter with the aid of  
which it is possible to set a risk which the resulting  
15 sequence of states has with respect to a prescribed  
state of the system.

2. The method as claimed in claim 1, in which a  
method of approximative dynamic programming is used for  
the purpose of determination.

3. The method as claimed in claim 2, in which the  
20 method of approximative dynamic programming is a method  
based on Q-learning.

4. The method as claimed in claim 3, in which the  
optimization function OFQ is formed within Q-learning  
in accordance with the following rule:

$$\text{OFQ} = Q(x; w^a),$$

25

- $x$  denoting a state in a state space  $X$
- $a$  denoting an action from an action space  $A$ , and
- $w^a$  denoting the weights of a function approximator  
which belong to the action  $a$ ,

30 and in which the weights of the function approximator  
are adapted in accordance with the following rule:

$$w_{t+1}^{at} = w_t^{at} + \eta_t \cdot N^k(d_t) \cdot \nabla Q(x_t; w_t^{at})$$

with the abbreviation

$$d_t = r(x_t, a_t, x_{t+1}) + \gamma \max_{a \in A} Q(x_{t+1}, w_t^a) - Q(x_t, w_t^{at})$$

- $x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,
- $a_t$  denoting an action from an action space  $A$ ,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t^{at}$  denoting the weighting vector associated with the action  $a_t$  before the adaptation step,
- $w_{t+1}^{at}$  denoting the weighing vector associated with the action  $a_t$  after the adaptation step,
- $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk monitoring parameter,
- $N^k$  denoting a risk monitoring function  $N^k(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- $\nabla Q(\cdot; \cdot)$  denoting the derivation of the function approximator according to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

5. The method as claimed in claim 2, in which the method of approximative dynamic programming is a method based on  $TD(\lambda)$ -learning.

25 6. The method as claimed in claim 5, in which the optimization function OFTD is formed within  $TD(\lambda)$ -learning in accordance with the following rule:

OFTD =  $J(\mathbf{x}; \mathbf{w})$

- $\mathbf{x}$  denoting a state in a state space  $X$ ,
- $a$  denoting an action from an action space  $A$ , and
- 5 •  $w$  denoting the weights of a function approximator and in which the weights of the function approximator are adapted in accordance with the following rule:

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \eta_t \cdot \mathbf{N}^*(\mathbf{d}_t) \cdot \mathbf{z}_t$$

10

with the abbreviations

$$\mathbf{d}_t = r(\mathbf{x}_t, a_t, \mathbf{x}_{t+1}) + \gamma J(\mathbf{x}_{t+1}; \mathbf{w}_t) - J(\mathbf{x}_t; \mathbf{w}_t),$$

$$15 \quad \mathbf{z}_t = \lambda \cdot \gamma \cdot \mathbf{z}_{t-1} + \nabla J(\mathbf{x}_t; \mathbf{w}_t),$$

$$\mathbf{z}_{-1} = 0$$

- $x_t, x_{t+1}$  respectively denoting a state in the state space  $X$ ,
- 20 •  $a_t$  denoting an action from an action space  $A$ ,
- $\gamma$  denoting a prescribable reduction factor,
- $w_t$  denoting the weighting vector before the adaptation step,
- 25 •  $w_{t+1}$  denoting the weighting vector after the adaptation step,
- $\eta_t$  ( $t = 1, \dots$ ) denoting a prescribable step size sequence,
- $\kappa \in [-1; 1]$  denoting a risk monitoring parameter,
- 30 •  $\mathbf{N}^*$  denoting a risk monitoring function  $\mathbf{N}^*(\xi) = (1 - \kappa \text{sign}(\xi))\xi$ ,
- $\nabla J(\cdot; \cdot)$  denoting the derivation of the function approximator according to its weights, and
- $r(x_t, a_t, x_{t+1})$  denoting a gain upon the transition of state from the state  $x_t$  to the subsequent state  $x_{t+1}$ .

7. The method as claimed in one of claims 1 to 6, in which the system is a technical system of which before the determination measured values are measured which are used in determining the sequence of actions.

5 8. The method as claimed in claim 7, in which the technical system is subjected to open-loop control in accordance with the sequence of actions.

9. The method as claimed in claim 7, in which the technical system is subjected to closed-loop control in  
10 accordance with the sequence of actions.

10. The method as claimed in one of claims 1 to 9, in which the system is modeled as a Markov decision problem.

11. The method as claimed in one of claims 1 to 10,  
15 being used in a traffic management system.

12. The method as claimed in one of claims 1 to 10, being used in a communications system.

13. The method as claimed in one of claims 1 to 10,  
being used to carry out access control in a  
20 communications network.

14. The method as claimed in one of claims 1 to 10, being used to carry out a routing in a communications network.

15. An arrangement for determining a sequence of  
25 actions for a system which has states, a transition in state between two states being performed on the basis of an action,

having a processor which is set up in such a way that  
the determination of the sequence of actions can be  
performed in such a way that a sequence of states  
resulting from the sequence of actions is optimized  
5 with regard to a prescribed optimization function, the  
optimization function including a variable parameter  
with the aid of which it is possible to set a risk  
which the resulting sequence of states has with respect  
to a prescribed state of the system.

10 16. The arrangement as claimed in claim 15, being  
used to subject a technical system to open-loop  
control.

17. The arrangement as claimed in claim 15, being  
used to subject a technical system to closed-loop  
15 control.

18. The arrangement as claimed in claim 15, being  
used in a traffic management system.

19. The arrangement as claimed in claim 15, being  
used in a communications system.

20. 20. The arrangement as claimed in claim 15, being  
used to carry out access control in a communications  
network.

21. The arrangement as claimed in claim 15, being  
used to carry out a routing in a communications  
25 network.

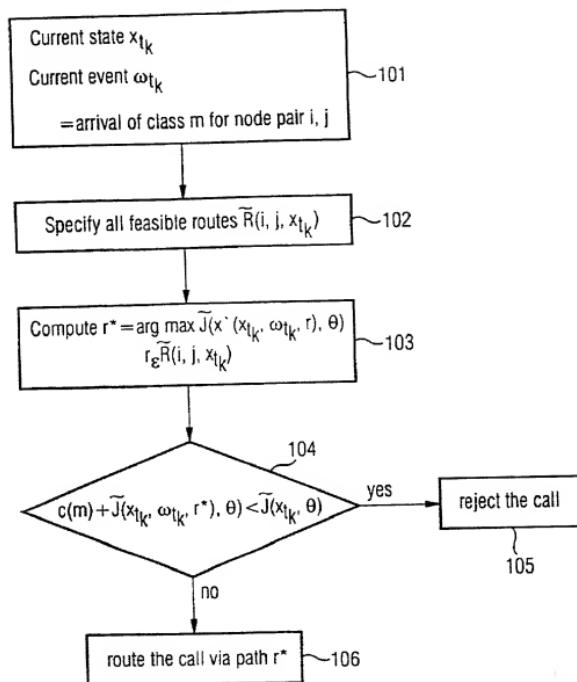
**Abstract**

**Method and arrangement for determining a sequence of actions for a system which has states, a transition in state between two states being performed on the basis of an action**

The determination of a sequence of actions is performed in such a way that a sequence of states resulting from the sequence of actions is optimized with regard to a prescribed optimization function. The optimization function includes a variable parameter with the aid of which it is possible to set a risk which the resulting sequence of states has with respect to a prescribed state of the system.

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FIG 1



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FIG 2

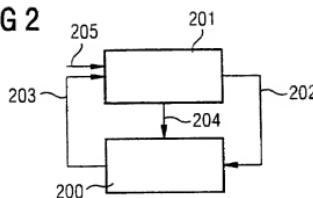
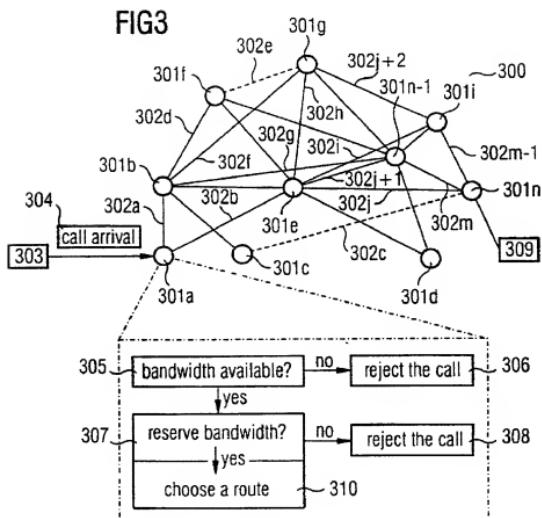


FIG3



04/787698 302 303 304 305 306 307 308 309

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FIG 4

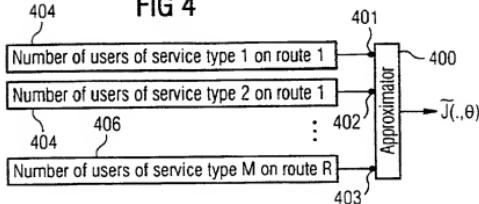
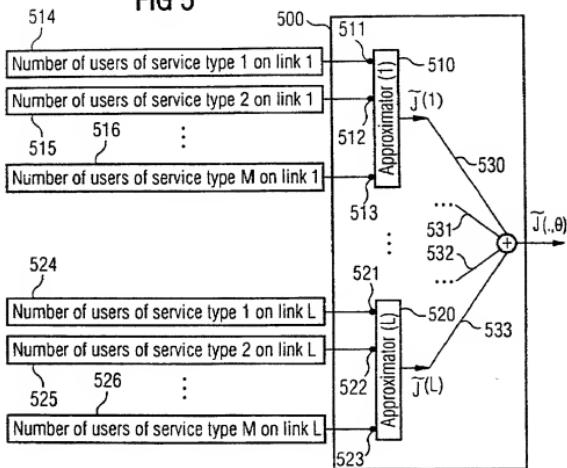
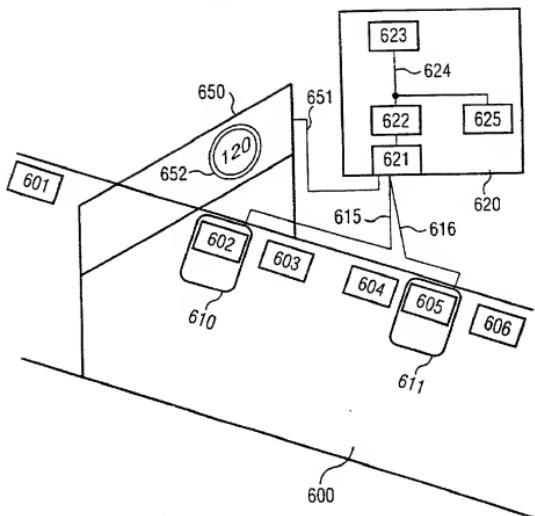


FIG 5



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FIG 6



**BOX PCT  
IN THE UNITED STATES DESIGNATED/ELECTED OFFICE  
OF THE UNITED STATES PATENT AND TRADEMARK OFFICE  
UNDER THE PATENT COOPERATION TREATY-CHAPTER II**

**CHANGE OF ADDRESS OF APPLICANTS' REPRESENTATIVE**

APPLICANT(S): RALF NEUNEIER, ET AL.  
ATTORNEY DOCKET NO.: P01,0020  
INTERNATIONAL APPLICATION NO.: PCT/DE99/02846  
INTERNATIONAL FILING DATE: 8 SEP 1999  
INVENTION: METHOD AND ARRANGEMENT FOR DETERMINING A SEQUENCE OF  
ACTIONS FOR A SYSTEM

Assistant Commissioner for Patents,  
Washington D.C. 20231

SIR:

Members of the firm of Hill & Simpson designated on the original Power of Attorney have merged into the firm of Schiff Hardin & Waite. All future correspondence for the above-referenced application therefore should be sent to the following address:

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**CUSTOMER NUMBER 26574**

Submitted by,

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**Declaration and Power of Attorney For Patent Application**  
**Erklärung Für Patentanmeldungen Mit Vollmacht**  
**German Language Declaration**

Als nachstehend benannter Erfinder erkläre ich hiermit  
an Eides Statt:

dass mein Wohnsitz, meine Postanschrift, und meine  
Staatsangehörigkeit den im Nachstehenden nach  
meinem Namen aufgeführten Angaben entsprechen,

dass ich, nach bestem Wissen der ursprüngliche,  
erste und alleinige Erfinder (falls nachstehend nur ein  
Name angegeben ist) oder ein ursprünglicher, erster  
und Miterfinder (falls nachstehend mehrere Namen  
aufgeführt sind) des Gegenstandes bin, für den dieser  
Antrag gestellt wird und für den ein Patent beantragt  
wird für die Erfindung mit dem Titel:

Verfahren und Anordnung zur Ermittlung  
einer Folge von Aktionen für ein System,  
welches Zustände aufweist, wobei ein Zu-  
standsübergang zwischen zwei Zuständen  
aufgrund einer Aktion erfolgt

deren Beschreibung

(zutreffendes ankreuzen)

hier beigefügt ist.

am \_\_\_\_\_ als

PCT internationale Anmeldung

PCT Anmeldungsnummer \_\_\_\_\_

Eingereicht wurde und am \_\_\_\_\_

Abgeändert wurde (falls tatsächlich abgeändert).

Ich bestätige hiermit, dass ich den Inhalt der obigen  
Patentanmeldung einschließlich der Ansprüche  
durchgesehen und verstanden habe, die eventuell  
durch einen Zusatzantrag wie oben erwähnt abgeän-  
dert wurde.

Ich erkenne meine Pflicht zur Offenbarung irgendwel-  
cher Informationen, die für die Prüfung der vorliegen-  
den Anmeldung in Einklang mit Absatz 37, Bundes-  
gesetzbuch, Paragraph 1.56(a) von Wichtigkeit sind,  
an.

Ich beanspruche hiermit ausländische Prioritätsvor-  
teile gemäß Abschnitt 35 der Zivilprozeßordnung der  
Vereinigten Staaten, Paragraph 119 aller unten ange-  
gebenen Auslandsanmeldungen für ein Patent oder  
eine Erfindersurkunde, und habe auch alle Auslands-  
anmeldungen für ein Patent oder eine Erfindersurkunde  
die nachstehend gekennzeichnet, die ein Anmelde-  
datum haben, das vor dem Anmeldedatum der An-  
meldung liegt, für die Priorität beansprucht wird.

As a below named inventor, I hereby declare that:

My residence, post office address and citizenship are  
as stated below next to my name,

I believe I am the original, first and sole inventor (if  
only one name is listed below) or an original, first and  
joint inventor (if plural names are listed below) of the  
subject matter which is claimed and for which a patent  
is sought on the invention entitled

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

the specification of which

(check one)

is attached hereto.

was filed on \_\_\_\_\_ as

PCT international application

PCT Application No. \_\_\_\_\_

and was amended on \_\_\_\_\_

(if applicable)

I hereby state that I have reviewed and understand the  
contents of the above identified specification, includ-  
ing the claims as amended by any amendment refer-  
red to above.

I acknowledge the duty to disclose information which  
is material to the examination of this application in  
accordance with Title 37, Code of Federal Regula-  
tions, §1.56(a).

I hereby claim foreign priority benefits under Title 35,  
United States Code, §119 of any foreign application(s)  
for patent or inventor's certificate listed below and  
have also identified below any foreign application for  
patent or inventor's certificate having a filing date  
before that of the application on which priority is clai-  
med:

## German Language Declaration

Prior foreign applications  
Priorität beansprucht

Priority Claimed

198 43 620.3	Germany	23.September 1998	<input checked="" type="checkbox"/> Yes Ja	<input type="checkbox"/> No Nein
(Number)	(Country)	(Day Month Year Filed) (Tag Monat Jahr eingereicht)		
(Nummer)	(Land)			

<input type="checkbox"/>	<input type="checkbox"/>	
(Number)	(Country)	(Day Month Year Filed) (Tag Monat Jahr eingereicht)
(Nummer)	(Land)	

<input type="checkbox"/>	<input type="checkbox"/>	
(Number)	(Country)	(Day Month Year Filed) (Tag Monat Jahr eingereicht)
(Nummer)	(Land)	

Ich beanspruche hiermit gemäss Absatz 35 der Zivilprozeßordnung der Vereinigten Staaten, Paragraph 120, den Vorzug aller unten aufgeführten Anmeldungen und falls der Gegenstand aus jedem Anspruch dieser Anmeldung nicht in einer früheren amerikanischen Patentanmeldung laut dem ersten Paragraphen des Absatzes 35 der Zivilprozeßordnung der Vereinigten Staaten, Paragraph 122 offenbart ist, erkenne ich gemäss Absatz 37, Bundesgesetzbuch, Paragraph 1.56(a) meine Pflicht zur Offenbarung von Informationen an, die zwischen dem Anmelde datum der früheren Anmeldung und dem nationalen oder PCT internationalen Anmelde datum dieser Anmeldung bekannt geworden sind.

I hereby claim the benefit under Title 35, United States Code, §120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, §122, I acknowledge the duty to disclose material information as defined in Title 37, Code of Federal Regulations, §1.56(a) which occurred between the filing date of the prior application and the national or PCT international filing date of this application.

(Application Serial No.)  
(Anmeldeseriennummer)

(Filing Date)  
(Anmelde datum)

(Status)  
(patentiert, anhangig,  
aufgegeben)

(Status)  
(patented, pending,  
abandoned)

(Application Serial No.)  
(Anmeldeseriennummer)

(Filing Date)  
(Anmelde datum)

(Status)  
(patentiert, anhangig,  
aufgegeben)

(Status)  
(patented, pending,  
abandoned)

Ich erkläre hiermit, dass alle von mir in der vorliegenden Erklärung gemachten Angaben nach meinem besten Wissen und Gewissen der vollen Wahrheit entsprechen, und dass ich diese eidesstattliche Erklärung in Kenntnis dessen abgabe, dass wissentlich und vorsätzlich falsche Angaben gemäss Paragraph 1001, Absatz 18 der Zivilprozeßordnung der Vereinigten Staaten von Amerika mit Geldstrafe belegt und/oder Gefängnis bestraft werden koennen, und dass derartig wissentlich und vorsätzlich falsche Angaben die Gültigkeit der vorliegenden Patentanmeldung oder eines darauf erteilten Patentes gefährden können.

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true, and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

### German Language Declaration

**VERTRETUNGSVOLLMACHT:** Als benannter Erfinder beauftrage ich hiermit den nachstehend benannten Patentanwalt(en) und/oder Patent-Agenten mit der Verfolgung der vorliegenden Patentanmeldung sowie mit der Abwicklung aller damit verbundenen Geschäfte vor dem Patent- und Warenzeichenamt. (Name und Registrationsnummer anführen)

**POWER OF ATTORNEY:** As a named inventor, I hereby appoint the following attorney(s) and/or agent(s) to prosecute this application and transact all business in the Patent and Trademark Office connected therewith. (list name and registration number)

Messrs. John D. Simpson (Registration No. 19,842) Lewis T. Steadman (17,074), William C. Stueber (16,453), P. Phillips Connor (19,259), Dennis A. Gross (24,410), Marvin Moody (16,549), Steven H. Noll (28,982), Brett A. Valiquet (27,841), Thomas J. Ross (29,275), Kevin W. Gwynn (29,927), Edward A. Lehmann (22,312), James D. Hobart (24,149), Robert M. Barrett (30,142), James Van Santen (16,584), J. Arthur Gross (13,615), Richard J. Schwarz (13,472) and Melvin A. Robinson (31,870), David R. Metzger (32,919), John R. Garrett (27,888) all members of the firm of Hill, Steadman & Simpson, A Professional Corporation.

And I hereby appoint

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(Name und Telefonnummer)

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\_\_\_\_

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A Professional Corporation  
85th Floor Sears Tower, Chicago, Illinois 60606

Voller Name des einzigen oder ursprünglichen Erfinders: <b>NEUNEIER, Ralf</b>		Full name of sole or first inventor:	
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<b>Bundesrepublik Deutschland</b>			
Voller Name des zweiten Miterfinders (falls zutreffend): <b>MIHATSCH, Oliver</b>		Full name of second joint inventor, if any:	
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Staatsangehörigkeit	Citizenship		
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<b>Bundesrepublik Deutschland</b>			

(Bitte entsprechende Informationen und Unterschriften im Falle von dritten und weiteren Miterfindern angeben).

(Supply similar information and signature for third and subsequent joint inventors).